

CENF-Common European Numeracy Framework

Common European Numeracy Framework

Literature Overview and Review

Kees Hoogland & Javier Díez-Palomar (Editors)

With contributions of:

Rosa Valls, Niamh O'Meara, Kathy O'Sullivan, Marina Stanic, Mieke van Groenestijn

Common European Numeracy Framework - Literature Overview and Review

© Utrecht, Januari 2022

Coordinator

HU University of Applied Sciences Utrecht, The Netherlands
Dr. Kees Hoogland, project manager

Partners

BFI-Oberösterreich, Linz, Austria
University of Limerick, Ireland
University of Barcelona, Spain



ISBN:

The terms and conditions of use are related to the Open Journal System and to Creative Commons Attribution License (CC-BY).



“The European Commission’s support for the production of this publication does not constitute an endorsement of the contents, which reflect the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained therein.”

Contents

Introduction	5
Executive Summary	7
Methods	9
Numeracy as a concept	11
Current trends in	
defining numeracy	17
Domains of numeracy	20
Definition and scope	21
Learning and teaching	25
Psychological factors	26
Vulnerable groups	27
Evaluation and assessment	28
Theories and frameworks	33
Contexts	36
Professional development	38
Institutional support	39
Challenges of numeracy	41
Numeracy as a social process	41
The impact of ICTs on numeracy	41
Critical citizenship	42
Teacher training programs	42
Assessment	42
References	43
Annex 1	60
Annex 2	73
Annex 3	75
Annex 4	77

Introduction

According to Coben et al., (2003) and O’Donoghue (2002) the term numeracy was created by the Crowther Report in 1959. Since then, the term has been intensively used, in a multiplicity of different meanings, evolving through the times from “basic arithmetic” towards a complex and multifaceted term that nowadays faces a plethora of old and new challenges, due to the increasingly mathematization of our everyday lives. There are innumerable reviews about numeracy (Coben, 2006b; Coben et al., 2003; Condelli, 2006; G. E. FitzSimons, 2004b; G. E. FitzSimons & Godden, 2000; Gal, 1993, 2000; Morton, McGuire, & Baynham, 2006; Tout & Gal, 2015), making almost impossible to cope with a shared consensual definition of what does it mean numeracy for everyone. This difficulty has been highlighted many times; hence we do not want to contribute with a new (and probably limited) review to the amount of previous works on that issue.

Instead, the objective of this literature review done within the frame of CENF is to open up the floor for contesting the new challenges that numeracy must face in the near future, acknowledging all the previous work done in the field. Numeracy has been one of the main concerns for governments around the World since 1950s. Citizens with STEM skills, able to deal with societies challenges, opening opportunities for development, is a condition that all countries want for their own citizens. Governments want a skilled workforce. They ask for detailed studies about the type of numeracy skills embedded within employments. APEL policies were developed by the EU in the 1990s to cope with such requirements.

Researchers have made visible the invisible. Numeracy can be invisible to people (Diez-Palomar, 2019; Steen, 2001, Wedege, 2010). But it can also be unpacked, tracked, and delivered to practitioners, decision-makers, teachers, to better train current and future learners. We need systematic reviews, to identify what are the key points, to pursue with our work in the field. The underlying report is a try to do it.

Despite that numeracy is a complex, multifaceted, concept, still many people tend to associate “numeracy” to basic arithmetic skills. Numeracy has been associated to do addition, multiplication, subtraction, division, basic numerical reasoning, and problem solving. Nothing more far than what numeracy and being numerate means. We are living in a numerate society, full of codes, data, uncertainty, and people do need to use numeracy skills to deal with all of that. Numeracy is one of the most fundamental skills, baseline for higher cognitive processes, but still, indispensable. International organizations have devoted (and still devote) huge resources to define, update and assess numeracy. This report and CENF, as a project, may contribute to expand this work and move forward with it.

Executive Summary

Numeracy is a multifaceted concept. This concept first appeared in the 1950s. *Numeracy* evolved from being associated to “basic arithmetic skills” towards a broader concept, referring to someone’s ability to interpret data and make connections that allow him or her to understand business, science and technology. In 1982 Stringer claimed that being numerate means “having an ‘at-homeness’ with numbers” and “the ability to have some appreciation and understanding of information which is presented in mathematical terms” (Cockcroft Report, 1982, p. 11). In 1994, during plenary session of the first conference of the *Adults Learning Mathematics* organization (ALM), Withnall discussed five different concepts of numeracy (as mathematics, aptitude, functional numeracy, numeracy in everyday life, and as context-specific). Numeracy moved towards a second phase in which it was conceptualized as “mathematics in context, or mathematics in everyday life.” Different authors defined numeracy in realistic situations, bringing together components such as “making meaning about mathematics in real everyday settings”, “mathematics in context”, “everyday numeracy thinking”, etc. In 1997 Tout also contributed to the international discussion introducing another dimension to the concept: being numerate means being critical. Gal, van Groenestijn, Schmitt, Tout and Clermont created the ALL Numeracy Group. They operationalized the term “numeracy” and introduced for the first time the term “numerate behavior”, involving five complexity factors (complexity of mathematical information/data, type of operation/skill, expected number of operations, plausibility of distractors, and type of match / problem transparency). In the 2000s the term numeracy evolved towards a more integrated concept with cultural, social, personal and emotional factors. Coben, O’Donoghue and FitzSimons (2000) published their *Perspectives on Adults Learning Mathematics*, an international handbook collecting the main contributions in ALM at that moment. In her chapter, FitzSimons organized her discussion concerning the adult learner of mathematics using a “macro- or institutional level” (including a social, cultural and political perspective), a “meso- or structural level”, and a “micro- or personal operational or subjective level.” Numeracy has been discussed in workplace contexts (nurses, cabinetmakers, farmers, warehouse pickers, etc.), in cultural settings, in social practices, etc. Current trends in defining numeracy suggest that it is a social practice (Yasukawa et al, 2018). Numeracy is historically and culturally situated.

Drawing on the literature review conducted within the frame of the CENF (Common European Numeracy Framework) project, numeracy can be defined as a multifaceted term integrating eight different domains: learning and teaching, psychology factors, vulnerable groups, evaluation and assessment, theories and frameworks, professional development, contexts, and institutional support. Numeracy is about knowing, but also about doing. As stated by Coben, “To be numerate means to be competent, confident, and comfortable with one’s judgements on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context.” From the learning and teaching point of view, there is a lack of literature on that issue, in the field of ALM. Drawing on the psychological factors, the main conclusion coming out from the review is that adults have a complex affective relationship with numeracy. If we consider the perspective of the vulnerable groups, some studies have reported cases of migrants, ethnic minorities (Latina families, Roma people, etc.), other women, etc., whose numeracy skills have been undervalued. From the evaluation and assessment standpoint, numeracy was taken as a serious thing by Governments all around the World. International surveys have tried to assess adults’ numerical

competences (IALS, ALL, PIAAC), defining levels of competence. The literature review also shows the diversity in terms of the theories and frameworks that have been used to discuss about numeracy (sociocultural approach, Freirean studies, Realistic mathematics, Dialogic Learning, Critical Studies, Women studies), although since numeracy is usually defined as a social practice, the most frequent theoretical frame are the so-called socio-cultural theories. In terms of “contexts”, there is a huge range of different contexts in which numeracy has been explored, including laboratory workers, fabrics, chemistry, automotive sector, forest guards, etc.), in banking situations (transactions, etc.), in the supermarket, shopping practices, a huge amount of studies related to diverse professions, like nursing practices, construction, carpentry, kiwifruit orchards, managing invoices, numeracy to become a critical citizen (reading graphs, understanding rates in an election procedure, etc.), commercials, using ICT, numeracy at home-practices, etc. However, drawing on a perspective in professional development, we noticed a lack of resources addressed to teach in service and pre-service teachers of ALM.

It seems that numeracy faces a number of challenges, for the coming future. The literature review suggests that there are five main types of challenges: numeracy as a social process, the impact of ICTs on numeracy, critical citizenship, teacher and training programs, and assessment.

Methods

There have been few literature reviews around the concept of *numeracy* since it was first used in the decade of the 1950s. Among the most remarkable cases is the work done by David Kaye, who compiled a set of definitions of *numeracy* in the last twenty years. He first presented a booklet collecting several definitions of numeracy at the annual meeting of the *Adults Learning Mathematics* (ALM) association, in Strobl (Austria), in 2003. Fifteen years later, he updated his original compilation in the ALM annual conference, held in London, in 2018. Diana Coben, Gail FitzSimons and John O'Donoghue have been working largely in the field, producing some of the most relevant references towards numeracy that we have in ALM. They co-edited a well-known handbook on ALM in the early 2000s (Coben, O'Donoghue, & FitzSimons, 2000). Another example of review of *numeracy* is the one done by Karaali, Villafane, Hernandez and Taylor (2016) who came with a review of the last fifty years about numeracy. Similarly, the *Numeracy Expert Group* reports for the PIAAC survey have provided informed reviews of the concept of numeracy along the last decades. Last year, in 2018, Katherine Safford-Ramus, John Keogh, John O'Donoghue and Terry Maguire presented a celebrating volume of the last 25 years of ALM, where some of the most relevant works about numeracy are compiled (Safford-Ramus, Keogh, O'Donoghue, & Maguire, 2018).

In all these cases, we see a similar methodology: authors report on a selected number of previous studies, about numeracy. They know the field; hence they are able to collect the main contributions and come with a summary of what numeracy means in the landscape of adults learning mathematics.

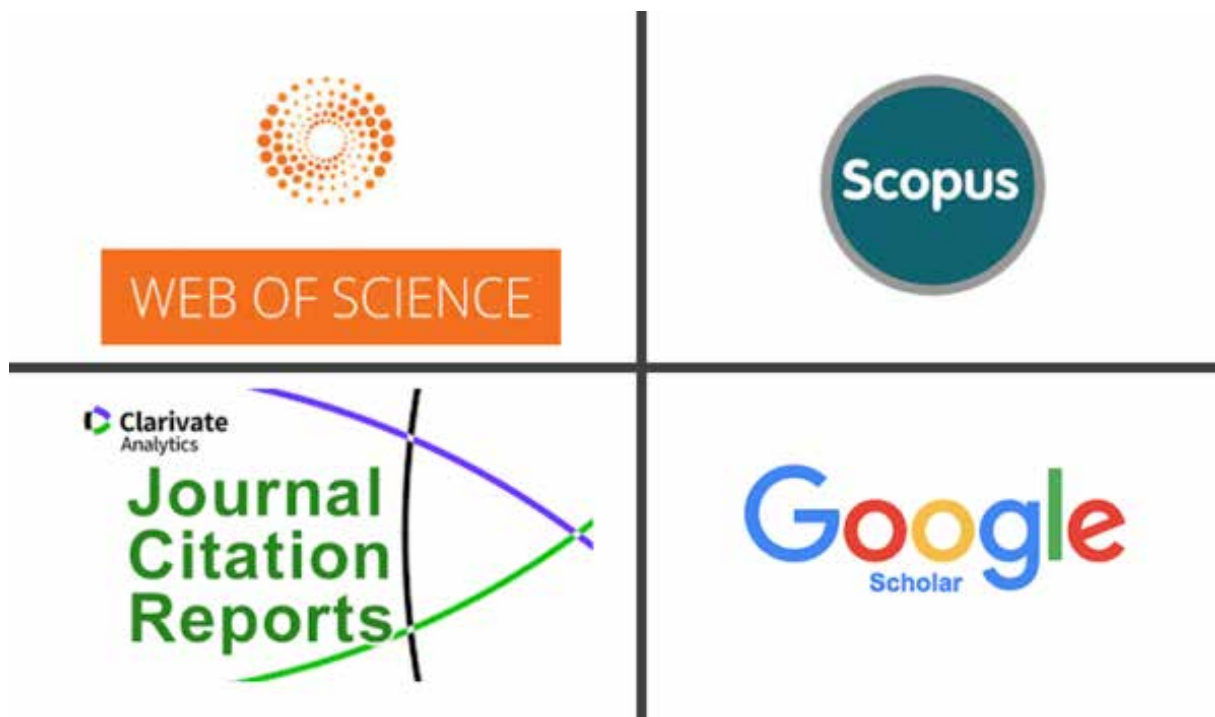
As Hart (2018) states, a literature review is,

The selection of available documents (both published and unpublished) on the topic, which contain information, ideas, data and evidence written from a particular standpoint to fulfil certain aims or express certain views on the nature of the topic and how it is to be investigated, and the effective evaluation of these documents in relation to the research being proposed.

In this report we use the same method. We have been using personal knowledge of the field, to select important articles. However, we have been also using the main scientific databases available, especially Web of Knowledge and Journal Citation Reports, as well as other major scientific databases such as Scopus and ERIC datafiles. Google Scholar has been used to collect data, that has been checked with the former databases for reliability.

Important references have been used, such as articles in scientific journals, book chapters and books about numeracy, PhD dissertations, reports, papers presented at national and international conferences, especially at the ALM annual meeting, which is the only international forum for researchers in ALM to present their work. Other international conferences were also used as sources of information, to find studies about numeracy, as ICME (there have been an active working group in this conference, which produced some of the handbooks available in ALM).

Figure 1. Main scientific databases used for the literature review.



The literature review has allowed us to identify what are the key sources in the field, what are the key theories, concepts or ideas surrounding numeracy, what are the epistemological and ontological grounds for the discipline, and what are the main questions and problems that have been addressed to date (and the future challenges that we face as a discipline). This literature review is also a proposal to structure and organize the main topics in the field.

Numeracy as a concept

Numeracy is a concept that appears in the scientific literature in the 1950s (Faragher & Brown, 2005). According to Coben et al., (2003), Kus (2018) and O'Donoghue (2002), the first time that numeracy was used was within the Crowther Report (1959).

When we say that a scientist is “illiterate,” we mean that he is not well enough read to be able to communicate effectively with those who have had a literary education. When we say that a historian or a linguist is “innumerate” we mean that he cannot even begin to understand what scientists and mathematicians are talking about. The aim of a good Sixth Form should be to send out into the world men and women who are both literate and numerate. (Crowther, 1959, p. 270, as cited in Kus, 2018, p. 59)

During the 1960s the term numeracy changed from being a “basic arithmetic” skill towards a broader concept, referring to someone’s ability to interpret data and make connections that allow him or her to understand business, science and technology. The definition moved from “knowing” to “doing.” In 1966, in an article published by *Economist*, Kus (2018) informs that we can find a similar concept to what we now understand by numeracy:

One of the early uses for “numeracy” is from an article in a 1966 issue of the *Economist*, stating that: “The need for numeracy today is enormous. Business requires people who have grasped the principles of reducing chaos of information to some kind of order” (OECD). (Kus, 2018, p. 59)

During the 1970s numeracy gained momentum. It was connected to the idea of educational credential (drawing on the *human capital* theories). In 1978 Wilfred Cockcroft was commissioned by the British Government to elaborate a survey on the teaching of mathematics in England and Wales. The Cockcroft Report (1982) defined numeracy as:

possessing an at-homeness with numbers and an ability to use mathematical skills to cope confidently with the practical demands of everyday life (Cockcroft Report, 1982, p. 11, as cited in Kus, 2018, p. 59).

In 1994 Alexandra Withnall wrote a paper titled “Towards a definition of numeracy”, presented at the 1st annual meeting of the Adults Learning Mathematics Association, held in Birmingham. Coben’s words introducing the proceedings of that meeting expressed the feeling of the scientific community towards numeracy:

We are long past the pioneering era, but numeracy is still the poor relation of adult literacy. What we need is the recognition of numeracy as a field worthy of serious study. (Coben, 1995, p. 3)

Withnall was one of the delegates participating at that conference. She acknowledged the use of numeracy but complained about the lack of consensus on a common definition for numeracy. She discussed numeracy as mathematics, as aptitude, as functional numeracy, as numeracy in everyday life, and as context-specific (see Table 1).

Table 1. Withnall's concepts of numeracy (1994)

Concept	Definition
Numeracy as mathematics	The minimum knowledge of mathematics and scientific subjects which any person should possess
Numeracy as aptitude	The possession of numerical skills as a special gift or talent.
Functional numeracy	A certain level of competence in numerical skills which is an essential pre-requisite both for effective individual participation in society and for the economic survival of that
Numeracy in everyday life	The possession of two particular attributes: (a) an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of everyday life; and (b) an appreciation and understanding of information presented in mathematical terms, e.g. in graphs, charts or tables or by reference to percentage increase or decrease.
Numeracy as Context-specific	Is a set of skills which can be applied in specific practical contexts, as the following: the goals (and values) of the activity within which it even makes sense to pose the problem; the social relations (including the exercise of power) in the setting within which the problem is posed; the material resources which form the basis for the activity.

Withnall was aware that numeracy was more than just "a set of mechanistic, discrete mathematical skills." Her testimony acknowledged the great debate around numeracy at that time, among the scientific community.

At the same conference, Jeff Evans and Ingrid Thorstad stood up for a definition of numeracy as context-specific, moving forward the traditional use of the term as equivalent to basic arithmetic skills.

Indeed, the idea of numeracy has been used to emphasise the need for "the maths" to be learned (and often used) in context. What distinguishes the context of everyday numerate thinking and problem solving from that of academic mathematics is the different activities and practices which form the different contexts. Thus, the numeracies used in the work of builders, pharmacists and shop-managers are all different - because they are based in different practices and hence are specific to them. (Evans and Thorstad, 1995, p. 64)

In 1997 Dave Tout contributed to the international discussion expanding the concept of numeracy, as more than just doing mathematics, or a narrow concept of functional mathematics. According to him,

We believe that numeracy is about making meaning in mathematics and being critical about maths. This view of numeracy is very different from numeracy being just about numbers, and it is a big step forward from numeracy or everyday maths that meant doing some functional maths. It is about using mathematics in all its guises - space and shape, measurement, data and statistics, algebra, and of course, number - to make sense of the real world, and using maths critically and being critical of maths itself. (Tout, 1997)

Tout was working with Iddo Gal, Yvan Clermont, Mieke van Groenestijn, Mary Jane Schmitt, and Myrna Manly as members of the ALL Numeracy Group. They redefined numeracy as a conceptual term to conduct international assessment studies, since numeracy was becoming a growing concern for many governments around the World. According to them,

We view numeracy as a complex, multifaceted and sometimes slippery construct. Our basic premise is that numeracy is the bridge that links mathematical knowledge, whether acquired via formal or informal learning, with functional and information-processing demands encountered in the real world. (Tout, 2000, p. 5)

Because their interest in operationalizing the term to conduct international surveys, the members of the ALL Numeracy Group created the concept “numerate behavior”, which involved five facets (see Figure 2).

Figure 2. Facets of the numerate behavior. (Tout, 2000, p. 5)

NUMERATE BEHAVIOR INVOLVES:

managing a situation or solving a problem

in a real context

everyday life

work

societal

further learning

by responding

identifying or locating

acting upon

interpreting

communicating about

to information about mathematical ideas

quantity & number

dimension & shape

pattern & relationships

data & chance

change

that is represented in a range of ways

objects & pictures

numbers & symbols

formulae

diagrams & maps

graphs

tables

texts

**and requires activation of a range of enabling
knowledge, behaviors, and processes**

mathematical knowledge and understanding

mathematical problem-solving skills

literacy skills

beliefs and attitudes.

They introduced the idea of “complexity-factors” to measure the levels of numeracy that someone has. The complexity factors included two types of aspects: mathematical and textual, as in Table 2.

Table 2. Complexity Factors – Overview

Aspects	Category	Range
Mathematical aspects	1. Complexity of mathematical information / data	Concrete / simple to abstract / complex
	2. Type of operation / skill	Simple to complex
	3. Expected number of operations	One to many
Textual aspects	4. Plausibility of distractors	No distractors to several distractors
	5. Type of match / problem transparency	Obvious / explicit to embedded / hidden

Source: (Tout, 2000, p. 6)

Contemporary to the ALL Numeracy Group, other scholars jumped into the discussion around numeracy. In 2001, Steen defined “quantitative literacy” as the capacity to:

Deal with quantitative aspects of life, and proposed that its elements included: confidence with mathematics; appreciation of the nature and history of mathematics and its significance for understanding issues in the public realm; logical thinking and decision-making; use of mathematics to solve practical everyday problems in different contexts; number sense and symbol sense; reasoning with data; and the ability to draw on a range of pre requisite mathematical knowledge and tools. (Steen, 2001 as cited in Goos, Dole, & Geiger, 2012, p. 3)

Similarly, (Hughes-Hallett, 2001) claims that numeracy involves dealing with quantitative information as a key aspect to understand our “data-drenched society.”

During those years Mieke van Groenestijn published her dissertation on numeracy (Van Groenestijn, 2002). She draw the attention to the huge variety of definitions around numeracy, inducing towards confusion within the field (Van Groenestijn, 2003). As a member of the ALL Numeracy team, she noticed how numeracy and literacy were at some point connected through the lenses of different frameworks, such as YALS, NALS or IALS.

After reviewing developments in UNESCO reports on adult education, the *Cockcroft Report*, studies conducted in Australia (Willis, 1990, Reeves, 1994), in USA as well (*Everybody Counts* report, YALS’ study on young adult literacy skills, the *National Adult Literacy Survey -NALS-*, the NCAL Report *-Issues and Challenges in Adult Numeracy-*), international surveys such as ALL (*Adult Literacy and Life Skills Survey*), PISA, and some other developments in Netherlands as well, van Groenestijn (2002) claimed that:

The essence of all definitions is that numeracy is about “mathematics embedded in a situation” in a very broad sense. Second, math is a “functional” part of real life situation that adults “have to manage”. In addition to these main topics, issues are mentioned as “feeling confident with numbers”, being able to “communicate” about mathematical topics, math as a tool for “describing and analyzing our world”, math for “effective functioning in one’s group and community”, math for

“critical and effective participation in a wide range of life roles” and the capacity to “further one’s own development”. (van Groenestijn, 2002, p. 36)

There was the shared assumption that numeracy was somehow connected (or embedded) with prose, documents and quantitative literacy. As Groenestijn (2003) cited:

The knowledge and skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials, such as balancing a checkbook, figuring out a tip, completing an order form, or determining the amount of interest on a loan from an advertisement. (OECD 1997; Dossey 1997; Houtkoop 1999, as cited in Van Groenestijn, 2003, p. 229)

In the ALL survey, conducted in 2002-2003, the term quantitative literacy was replaced by numeracy, in order to develop assessment items. The definition introduced a “competence” component, as the Cockcroft (1982) report did twenty years earlier (1982). Numeracy in the ALL Survey was defined as:

The knowledge and skills required to effectively manage the mathematical demands of diverse situations. (Van Groenestijn, 2003, p. 230)

In her article, Groenestijn (2003) highlighted the fact that this definition uses the word “manage”, meaning that being numerate involves more than just knowing mathematics; instead, it means also what, later on in the context of the PIAAC, has been called “doing”.

The word “manage” indicates that being numerate encompasses more than just knowing mathematics. It implies that to organize their lives as individuals, as workers, and as citizens, adults need to feel confident of their own mathematical capacities and be able to make effective decisions in mathematical situations in real life. (Van Groenestijn, 2003, p. 230)

Living in a continuously changing society Van Groenestijn defined numeracy as a dynamic concept. Numeracy changes over time in line with changes in our society. As a member of the ALL Numeracy team she elaborated the ALL numeracy definition a bit more and defined numeracy as:

Numeracy encompasses the knowledge and skills required to effectively manage the mathematical demands in personal, societal and work situations, in combination with the ability to accommodate and adjust flexibly to new demands in a continuously rapidly changing society that is highly dominated by quantitative information. (Van Groenestijn, 2002, p. 37)

More or less at the same time, in 2003, David Kaye introduced a review about the concept of numeracy at the 10th Annual Meeting of the Adults Learning Mathematics Association, in Strobl, Austria. He collected some of the most relevant definitions of numeracy in the literature, available at this moment. Among all of them, Kaye included the definitions by O’Donoghue (2003), Maguire and O’Donoghue (2004), Groenestijn (2002), Tickly and Wolf (2000), Coben (2000, 2002), Withnall (1995), Johnson and Johnson (2002), Benn (1997), the one provided within the Cockcroft report (1982), and some other more. Drawing on those definitions, Kaye was able to argue that numeracy has evolved from a narrow-view based on basic arithmetic, aligning “numeracy” to “being able to use basic arithmetic skills (addition, subtraction, multiplication and division) as well as conduct and solve problems according to those skills”, towards a more sophisticated definition, in which numeracy is presented as a compilation of more complex cognitive skills.

Kaye cited O’Donoghue (2003) to cope with this evolution of the scope of the term:

... may signify any one of a number of things including, basic computational arithmetic, essential mathematics, social mathematics, survival skills for everyday life, quantitative literacy, mathematical literacy and an aspect of mathematical power. These descriptions span a spectrum of personal abilities from basic skills to high-level cognitive abilities such as problem solving and communication (O’Donoghue, 2003, as cited in Kaye, 2003, p. 194)

Drawing on this analysis, Kaye claimed that numeracy implies “math plus”, meaning that numeracy involves “social situations”, “individual attitudes and emotions”, “vocational education and training”, “political and economic planning”, and so forth. Then, he moved towards what he called “big numeracy” and “small numeracy”:

‘Big Numeracy’ is the numeracy that is larger than mathematics -it is ‘mathematics plus’. It has no reference (necessarily) to a level of skill, technique or analysis, but it always includes some reference to a context; it is relevant and relative.

‘Small Numeracy’ in contrast refers very literally to working with numbers; it implies that the level of skills and techniques is simple and could be thought of as little more than arithmetic. (Kaye, 2003, p. 196)

Kaye’s paper prompted to consider numeracy as something more than just (basic) arithmetic. He shared views as the one by Tickly and Wolf (2000):

The word ‘numeracy’ is not, outside educational circles, a commonly used or recognized term... Government policy, however, is currently overwhelmingly concerned with numeracy in the sense of arithmetic and number sense... But numeracy -in its current meaning of arithmetic and number sense- is not the same as mathematics. If we imply it is, or that only numeracy is needed by students, we do them, and our whole society, a grave disservice... This is not just because the learning of mathematics piles up more and more skills and techniques (although it does that too), but because it transforms people’s ability to conceptualize and structure relationships, to model the world and to change it. (Tickly and Wolf, as cited in Kaye, 2003, p. 196)

Several years later, in 2010, Kaye re-visited his work on defining numeracy at the 17th Annual Meeting of ALM, in Oslo; and again, in 2018, in London, at the 25th Annual Meeting. His work was compiled as a chapter at the ICME Monograph series *Contemporary research in adult and lifelong learning of mathematics*, edited by Katherine Safford-Ramus, Jürgen Maaß and Evelyn Süss-Stepancik. Quoting Tout (1997), Kaye claims that “numeracy is not less than mathematics but more.” (Tout, 1997, as cited in Kaye, 2018, p. 12) Others have made the same statement. (Johnston & Tout, 1995; Yasukawa, Johnston, & Yates, 1995)

In the same vein, official institutions such as the *Australian* Department of Education and Early Childhood Development in Victoria have defined numeracy as different from mathematics:

Numeracy is not the same as mathematics, nor is it an alternative to mathematics. Rather, it is an equal and supporting partner in helping students learn to cope with the quantitative demands of modern society. Whereas mathematics is a well-established discipline, numeracy is necessarily interdisciplinary. Like writing, numeracy must permeate the curriculum. When it does, also like writing, it will enhance students’ understanding of all subjects and their capacity to lead informed lives. (Department of Education and Early Childhood Development in Victoria, Australia, 2009, p. 8, as cited in Kus, 2018, p. 60)

Current trends in defining numeracy

Yasukawa, Rogers, Jackson, and Street (2018) start their book on numeracy noticing that this concept is still a “slippery concept”, as Coben and her colleagues stated in 2003. We did an overview of the most relevant facts in the evolution of the definition for numeracy, since it was used for the first time in 1959. Yet, there is a lack of a shared and universal definition of numeracy, although probably everyone would have a roughly idea of what does it mean when hearing the word. However, what is out of question, is the fact that numeracy has evolved from being just a basic-arithmetic concept, towards a broader and more extensive concept, referring to a range of different competences and skills. Current reviews and studies within the field notice that fact, especially fueled by the fact that we are living in an increasingly technological and big-data-based society, in which numerical competences become the more and more mandatory for everyone to be “included.”

However, as Yasukawa, Rogers, Jackson, and Street (2018) notice, teaching numeracy (curriculum, teaching strategies, activities development, etc.) is still highly influenced by the narrow view of numeracy as a basic arithmetic skill. Current trends in defining numeracy make unavoidable the fact of re-defining professional teachers’ development in the field of ALM and, more concretely, in terms of teaching numeracy in adult educational centers.

According to (Geiger, Forgasz, & Goos, 2015), numeracy nowadays is used to:

identify the knowledge and capabilities required to accommodate the mathematical demands of private and public life and to participate in society as informed, reflective, and contributing citizens (...) extends beyond the mastery of basic arithmetic skills to how to connect the mathematics learnt in formal situations, such as school classrooms, to real world problems....[It] involves the capability to: make sense of non-mathematical contexts through a mathematical lens; exercise critical judgement; and explore and bring to resolution real world problems. (Geiger, Forgasz, & Goos, 2015, p. 531)

The Numeracy Expert Group working at the ALL survey introduced the term numerate behavior (Gal et al. 1999) which refers to the ability of using mathematical information and ideas.

Numerate behavior involves managing a situation or solving a problem in a real context by responding to mathematical information that is represented in a range of ways and requires the activation of a range of enabling processes and behaviors.” (Gal et al., 1999, as quoted in Van Groenestijn, 2002, p. 31)

It involves using this capacity for practical, personal and work-related purposes, to interpret mathematical and graphical information for societal purposes, and for utilizing and applying mathematical knowledge. According the them, numeracy is:

the ability to access, use, interpret and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life. To this end, numeracy involves managing a situation or solving a problem in a real context, by responding to mathematical content/information/ideas represented in multiple ways. (OECD, 2013, p. 18)

In a recent book written by (Jurdak, 2016), he states that numeracy has to be understood (and defined) within the realm of real mathematics. According to him, numeracy covers the space between the mathematics practiced in schools and mathematics as enacted in social and workplace settings. He uses the work “literacy”, connecting it to mathematics. He draws on the approach that [mathematics] literacy is socially grounded. This means that we need to understand mathematics literacy within a social system of practices. He states that:

In mathematical literacy, mathematics is the symbolic system and the external world of referents is some aspects of the culture, mainly those that relate to quantity, space, and measurement. (Jurdak, 2016, p. 37)

This definition is connected to the term “mathematical literacy” used by OECD/PISA.

Mathematical literacy is an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen. (OECD, 2003, p. 24)

That definition was a result of the IALS Survey (precursor of the ALL Survey), that was borrowed by PISA. From his point of view, mathematics literacy is always embedded within social and cultural practices, hence we cannot separate it from that type of referents. This also matters in terms of what we consider to be literacy and mathematically literate., which obvious impact in curriculum within ALM courses, for instance.

Drawing on Jablonka’s (2003) approach, Jurdak (2016) discusses the meaning of mathematical literacy. According to Jablonka (2003), mathematical literacy comprises five different perspectives: mathematical literacy for developing human capital, mathematical literacy for cultural identity, mathematical literacy for social change, mathematical literacy for environmental awareness, and mathematical literacy for evaluating mathematics. Jurdak (2016) claims that there are four domains for mathematics literacy: autonomous literacy, ideological literacy, critical literacy, and literacy as a cultural capital. He defends that mathematical literacy is a multidimensional concept, being transversal to school subjects (curriculum), which, at least for his point of view, is not applied in a critical way (as Freire was envisioning education and literacy). In this sense, it seems very clear that for him mathematics literacy is a way of being critical in our real world, which is social al culturally situated. Numeracy, drawing on this approach, is more connected to a social approach, rather than being only “basic mathematics”, as some educational stakeholders have defended over the years.

Similarly, in a more recent book, Yasukawa and her colleagues (2018) define numeracy as a social practice. According to them, this is probably a consequence of the influence of the socio-cultural theories within the mathematics education field. Mathematics, mathematical practices, curriculum, teaching, have been re-defined in terms of cultural-historical activities. Concepts such as situated practices on cognition, dialogic learning, ethnomathematics, ZPD spaces, distributed cognition, are colonized during the last years the field of mathematics education, making obvious that teaching and learning is a social-based practice; thus, numeracy (as well as other concepts) need to be re-defined in such terms.

Citing different studies, Yasukawa et al. (2018) claim that:

Studies like these laid the foundation for establishing a situative perspective on cognition, in which “[c]ognition” observed in everyday practice is distributed – stretched over, not divided among – mind, body, activity and culturally organized settings (which include other actors)’ (Lave 1988, 1) and learning is represented by change in participation in an activity setting over time (Lave and Wenger 1991). (Yasukawa et al., 2018, p. 6)

Thus, numeracy is historically and culturally situated. Additionally, they use CHAT theory to also highlight the fact that numeracy is also embedded in VET settings, as multiple previous research has pointed out (CREA, 1993; FitzSimons, 2005; Kanes, 2002; Pozzi, Noss, & Hoyles, 1998; Roth, 2012; Triantafillou & Potari, 2006; Julian Williams & Wake, 2007).

Yasukawa and her colleagues (2018), based on previous studies (D. Baker, 1998, 2008), conclude that in fact, numeracy is a social practice, as stated by different scholars, such as (D. Baker, Street, & Tomlin, 2003; Street, Baker, & Tomlin, 2005; Yasukawa et al., 2018), who coined the concepts of numeracy events and numeracy practices.

Domains of numeracy

In this section we develop the coding scheme that we have created drawing on the review and analysis of the previous studies published in peer-review journals, books and proceedings over the last years, on numeracy. As explained in the methodological section, we have followed a method similar to the one used within the grounded theory, which is “create theory” at the same time as we review the previous studies, getting engaged in an interactive process in which we exchanged this review with several experts in the area, to include also their thoughts in creating the codes.

Next table includes the final (current) version of the codes used to organize and review the contributions from the domain.

Table 3. Coding scheme used to conduct the review of the literature

Code	Number of items
Definition	
Scope	
Lifelong learning	
Learning and Teaching	
Dialogic practices	
Curriculum and instruction	
Episodes of case studies	
Classroom practices / methods	
Resources	
Psychological factors	
Affect (including math anxiety)	
Agency and self-efficacy	
Resilience	
Motivation	
Cognition	
Vulnerable groups	
Migrants, refugees	
Ethnic minorities	
Women	
Difficulties	

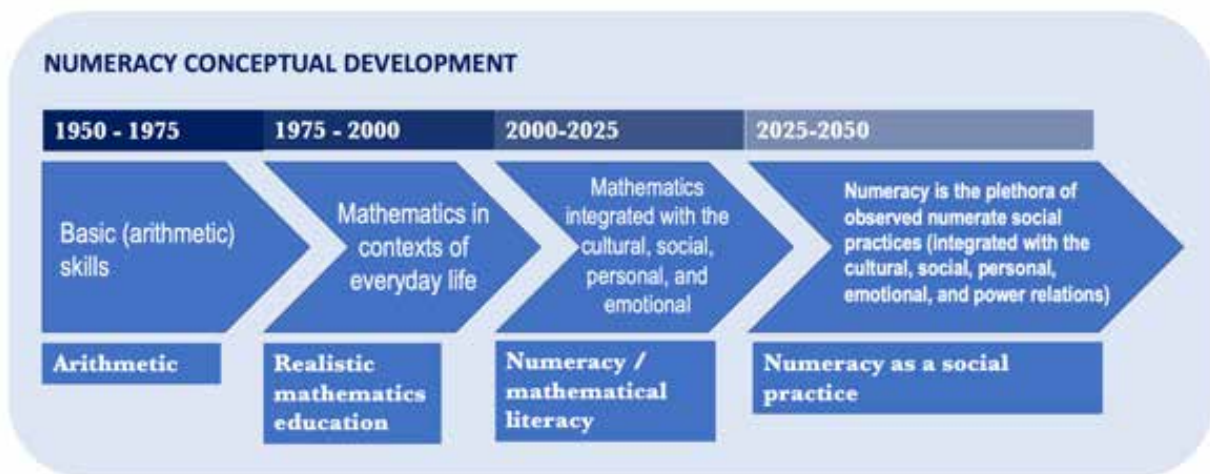
Evaluation and assessment
Theories and frameworks
Socio-cultural approaches
Paulo Freire
Realistic mathematics
Dialogic learning
Critical studies
Women studies
Professional development
Relevant competences
Examples
Resources (lesson study, didactic suitability, etc.)
Models (PCK, MTSK, CMF, APOS, EOS, noticing,
Impact
Contexts
Non-formal and informal education (contexts)
Health literacy
Financial literacy
Workplace
Domestic work
Statistics literacy
Learning transfer
Institutional support
Policies
Adult educational systems

Drawing on the comments and contributions from the participants in the literature review, a number of “big categories” were created (highlighted in the Table 3). Each of them was breaking down into more specific subcategories, covering a total of 47 categories. In the next pages, we provide an overview for each of the main categories, and their respective sub-categories.

Definition and scope

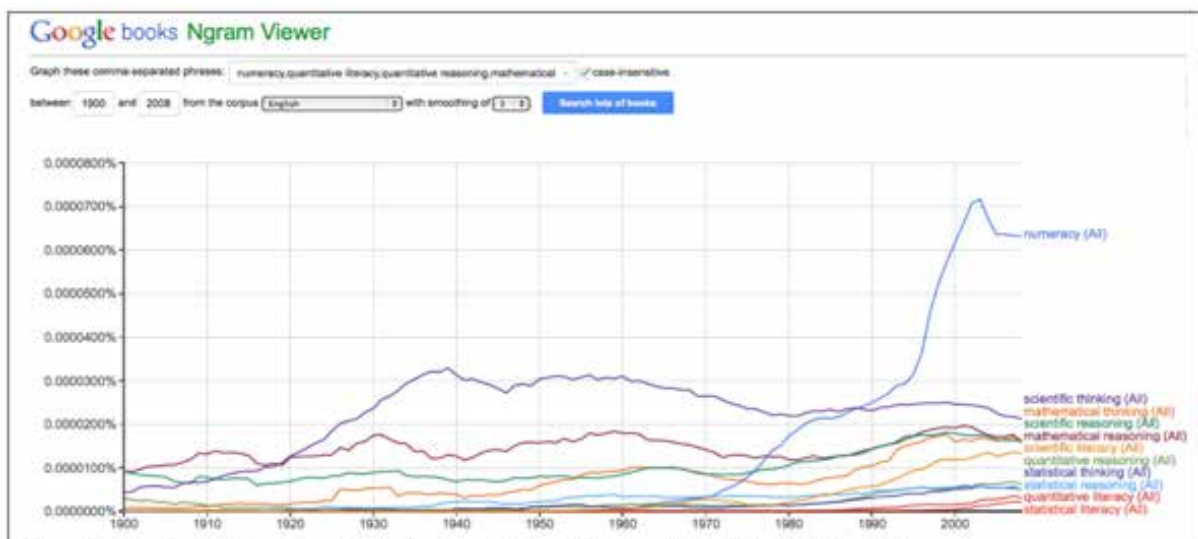
As we have noticed in the previous sections, there is no a single, consensual, definition for numeracy. The use of this word has changed along the time, from the late 1950s up to now. Numeracy has gained presence in front of other terms, such as “quantitative literacy”, “quantitative reasoning”, etc. (Karaali, 2016) Figure 3 summarizes the main components in that evolution of the term.

Figure 3. Numeracy conceptual development



According to Karaali (2016), numeracy is, largely, the most used term in the literature review (Figure 4).

Figure 4. Prevalence of the use of different terms in books, over the time. Source: Karaali, 2016, p. 4



Although there is no common agreement on what numeracy means, when we distinguish it from mathematics, we can find a wide consensus on the fact that numeracy is not mathematics. There are many authors supporting this claim (Arney, 2002; Barwell, 2004; Callingham, Beswick, & Ferme, 2015; O'Donoghue, 2003). In the next table we compile some relevant quotes endorsing this idea:

Table 4. Definition of numeracy related to mathematics

Numeracy as distinct from mathematics	Unlike mathematics, which is primarily about a Platonic realm of abstract structures, numeracy is often anchored in data derived from and attached to the empirical world. (Arney, 2002, p. 5)
	Numeracy is different from mathematics. Mathematics is abstract, absolute and ideal. It is about creating tools and models for understanding concepts. Numeracy is concrete and contextual; it deals with real things in ways specific to their context. (Callingham, Beswick, & Ferme, 2015, p. 557)
	Mathematics and numeracy are not congruent. Nor is numeracy an accidental or automatic by-product of mathematics education at any level. When the goal is numeracy some mathematics will be involved but mathematical skills alone do not constitute numeracy. (O’Donoghue, 2003, p. 8)
	Numeracy somehow involves the use of numbers, calculation or diagrams in social practice, whilst mathematics involves some degree of abstraction or concern with structure (Barwell, 2004, p. 20)

As displayed in Figure 3, numeracy has been associated for many years to “basic arithmetic”, unless the recommendations from the international scientific community that this is not true and we cannot assume that numeracy means “basic arithmetic.” (Gal et al., 2009; Gal & Tout, 2014; Hoogland, Díez-Palomar, & Maguire, 2018; Tout et al., 2017)

However, this conceptual link with the idea of “knowing” is also embedded in many definitions, bridging numeracy with quantities, and numbers (in opposition to another relevant trend in the field, with associate numeracy with the idea of “doing”). This is the case for terms such as “quantitative literacy”, for example.

Figure 5. Components of the numeracy definition



Table 5. Quotes illustrating the different ideas / concepts of numeracy, according to the knowing/doing distinction.

<p>Numeracy as “knowing”</p>	<p>Numeracy is also known – particularly in the United States – as ‘quantitative literacy’. In that country’s 1992 National Adult Literacy Survey (NALS), quantitative literacy was defined as ‘the knowledge and skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials, such as figuring out a tip, completing an order form, or determining the amount of interest on a loan from an advertisement’ (Hector-Mason, et al, 2006). (Carpentieri & Frumkin, 2010, p. 6)</p>
<p>Numeracy as “doing”</p>	<p>(...) the ability to understand, use, calculate, manipulate, interpret results and communicate mathematical information (DfES, 2001, as cited in Carpentieri & Frumkin, 2010, p. 6)</p> <p>To be numerate means to be competent, confident, and comfortable with one’s judgements on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context. (Coben, 2000b:35; emphasis in the original) (Coben et al., 2003, p. 10)</p> <p>Numeracy is that of ‘mathematical literacy’ used by the Programme for International Assessment (PISA) [Organisation for Economic Co-operation and Development (OECD) 1999]. Mathematical literacy is defined as “Identifying, understanding and engaging in mathematics and making well-founded judgements about the role that mathematics plays, as needed for an individual’s current and future life as a constructive, concerned and reflective citizen” (OECD 1999, p. 12, as cited in Callingham, Beswick, & Ferme, 2015, p. 549)</p> <p>(...) numeracy is not so much about understanding abstract concepts as about applying elementary tools in sophisticated settings. (Steen, 2001, p. 108)</p>

Numeracy has been also associated towards using mathematics in everyday, real, contexts (Cooke, 2017; Gal et al., 2009; Gal, Van Groenestijn, Manly, Schmitt, & Tout, 2005; Manly, Tout, Van Groenestijn, & Clermont, 2000).

In recent years the international scientific community as reach a certain consensus around the idea that numeracy must to be understood as a social practice, rather than a set of quantitative contents (numbers, symbols, etc.), or procedures (addition/subtraction structures, multiplication/division structures, proportional reasoning, etc.) (Baker et al., 2003; Barton & Hamilton, 2012; Callingham et al., 2015; Yasukawa et al., 2018). The next quotes illustrate this fact, which is now a trend in the field:

are not only the events in which numerical activity is involved, but [...] the broader cultural conceptions that give meaning to the event [...] (Baker et al., 2003, P 12)

Numeracy is primarily something people do; it is an activity, located in the space between thought and text. Numeracy does not just reside in people's heads as a set of skills to be learned, and it does not just reside on paper, captured as texts to be analyzed. Like all human activity, numeracy is essentially social, and it is located in the interaction between people (Barton & Hamilton, 2012, p. 3)

The nature of numeracy implies that both mathematics and social context are involved. In the Australian curriculum, numeracy is included as one of seven general capabilities that it is the responsibility of every teacher to develop in their students. Many teachers, however, dislike or actively avoid mathematics. This situation tends to lead to opportunities for numeracy development in school students to be overlooked or ignored. A key issue is then to help teachers to recognise the appropriate mathematical ideas in subjects other than mathematics, and then to suggest ways in which these ideas can be acknowledged and developed in context. Three different approaches to working with both pre- and in-service high school teachers are described, and the outcomes discussed in the context of building professional capital. Findings suggest that teachers do not recognise the complexity of numeracy, and benefit from supportive professional learning developing professional capital with respect to numeracy (Callingham et al., 2015)

What do we mean by "numeracy as social practice"? There is a growing body of research that has suggested that, similarly to literacy, what counts as numeracy and what it means to engage in numeracy cannot be understood apart from the social, cultural, and political contexts in which the mathematical activity is embedded (e.g. Harris, 1991; Hoyles, Noss, Kent, & Bakker, 2010; Lave, 1988; Mukhopadhyay & Roth, 2012; Nunes, Schliemann, & Carraher, 1993; Reder & Brynner, 2009; Saxe, 1991; Street, Baker, & Tomlin, 2005). The symposium brings together researchers who adopt a social practices approach to their research in numeracy, and those who are interested in the implications of this approach to numeracy for research, pedagogy, and policy. (Yasukawa et al., 2018)

There are also a number of authors claiming that numeracy also has to be understood as a competence to become critical thinker in our society, meaning that individuals must use numeracy skills to confront critically our world, making informed decisions, and been able to fully participate in democratic societies (Evans, Ruane, & Southall, 2019; Evans, Wedege, & Yasukawa, 2012; Geiger, 2019; Geiger, Forgasz, et al., 2015; Lucas, 2019; Volchok, 2019).

Learning and teaching

Although there is a decent number of studies on numeracy during the last decades, when we have a look on them from the teaching and learning point of view, then we realize that this is still a domain that needs to be expanded in the future.

In terms of learning, many of the studies conducted in the field draw on Adult Education approaches towards how adults learn (R. Flecha, 2000; Jarvis, 2004; Mezirow, 2000). A vast majority of them refers to Freire and Freirean theories of instruction (Freire, 1970a, 1970b, 2000; Tygel & Kirsch, 2016). Numeracy is seen as a practice of freedom, as a way to become critically included in our society (Díez-Palomar, 2017).

However, when we move towards "teaching", there is a lack of literature on that issue, in the field of ALM. Almost all references are children-focused, with some remarkable exceptions (Coben & Chanda, 2000; Gal, 2002; Galligan & Taylor, 2008; Swain, 2005; Nicol & Anderson, 2000).

When we look in the literature review, the most common evidence found in teaching numeracy to adults is the fact that the successful actions use learners' experience as a resource to teach them numeracy. Street, Rogers, and Baker (2006) for instance, explain how a group of teachers in India use participants' experiences to make sense to the numeracy within the curriculum. They use familiar situations for the participants, creating scenarios for discussing numeracy. Similarly, Swain, Baker, Holder, Newmarch, and Coben (2005) draw on the daily application of numeracy

to make it meaningful for adults to learn. According to Galligan and Taylor (2008), holistic approaches to teaching are the most common strategy that teachers in ALM use to teach numeracy.

According to some studies, this is a way to motivate adults to learn (Swain et al., 2005; Windisch, 2016). But the literature suggests that more than a way to motivate adults, the fact is that using personal stories, experiences, memories, etc., makes more sense to adults, who are able to connect the mathematical objects embedded within numeracy, with their experiences, making sense to them, and, thus, expanding their understanding on them.

This is consistent with the usual way to work with adults in adult education. This field is highly influenced by Freire's work (Freire, 1970a, 1970b, 2000; Tygel & Kirsch, 2016), who demonstrated that familiar context and in-context practices produce better learning results among adults, than school-like and de-contextualized strategies. Dialogic teaching has proven to be particularly successful in teaching adults (R. Flecha, 2000). This has been also proven in ALM (Díez-Palomar, 2020).

Resources include personal narratives, familiar situations (Street et al., 2006), dialogic mathematics gatherings (Díez-Palomar, 2017), everyday situations, etc. Most of the times, teachers become "researchers" collaborating with adults to explore together the topics been discussed (Street et al., 2006).

Psychological factors

The most common situation reported through the literature review among adult who are confronted with numeracy is fear (Breen, 2003; Gustafsson, Mouwitz, & Turner, 2004; Southwood, 2011). Many adults feel panic when they are challenged with numeracy. However, the review reveals that this interpretation is, somehow, quite simplistic, since adults also accumulate a variety of opposed feelings towards mathematics and numeracy. Respect, appreciation, valorization, fear, hope, ... adults use to declare that numeracy (and mathematics) is very important in our lives, but it is also a tricky subject, demanding, that may make them feel afraid of not being able to answer the questions raised in the classroom (Swain et al., 2005). The main conclusion coming out from the review is that adults have a complex affective relationship with numeracy.

In extreme cases, some adults may feel anxious towards numeracy. "Math anxiety" is a well reported emotion through the research (Andrews & Brown, 2015; Evans, 2002; Handler, 1990; Jameson & Fusco, 2014; Klinger, 2006, 2010; Maloney & Beilock, 2012), from many decades now (Stent, 1977).

In the same vein, adults' resilience is also a well-known feeling (Alkema, 2019; Cousins, Brindley, Baker, & Johnston-Wilder, 2019; Strümpfer, 1999; Tett & Maclachlan, 2007), as self-confidence (FitzSimons, 2008; Jameson & Fusco, 2014; Tett, Maclachlan, Hall, Edwards, & Garside, 2006; Yasukawa, Widin, & Chodkiewicz, 2008). Adults who experience positive episodes of mathematics learning, whose numeracy abilities are recognized and valued by the group, may develop a sense of resilience and may enhance their self-confidence. This is consistent throughout the literature reviewed. Thus, teachers must find ways to create this type of situations. According to the evidence, drawing on adults' everyday experience is a successful strategy to create such type of positive environment, which will empower adults.

An important aspect emerging from the literature review is the need to distinguish between adults (individuals who never went to school, or with low-level school background; specially women), and young adults (defined as men and women in their twenties, who dropped school).

Young adults present a remarkable and persistent feeling of rejection and resistance towards numeracy, because they use to connect this idea to their experience with school (which is usually negative). Numeracy, in this case, use to be confused with “mathematics.” They use to engage in VET or HE programs (Bloom, 2010; Mac Iver & Mac Iver, 2009).

Vulnerable groups

There is limited work done in the field about vulnerable groups and numeracy. Although we can find indirect references to the idea of vulnerability, as “social exclusion” in terms of not being able to access the labor market, not being able to fully participate in our society (i.e. banking, shopping, using ICTs, etc.), lack of knowledge to make decisions, based on critical thinking and strategic reasoning, etc., the issue of vulnerability, per se, has been missed among the literature in ALM and numeracy. It seems that overall we assume that innumeracy is somehow attached to a factual situation of vulnerability. The only special issue in a scientific journal (ZDM) on numeracy and vulnerability has been published in 2020. This issue, edited by Iddo Gal, Anke Grotlüschen, Gabriele Kaiser, defines vulnerability as:

the WHO and public health sciences introduced it, relates to populations which are systematically exposed to factors (e.g., lack of clean water, unsafe living or working conditions, poor nutrition) that increase their risk to be sick, reduce life expectancy, slow physical development or growth, etc. Seen in this way, vulnerability is not a property of people but depends on the life circumstances and social conditions (Quenzel et al. 2016; Shi et al. 2008) which can be changed. (Gal, Grotlüschen, & Kaiser, 2020)

They also use the term “vulnerability” to refer to populations who are exposed towards physically or mentally impaired due to ageing, or at-risk of discrimination (because being immigrants, refugees, or due to religious differences). FitzSimons, Grotlüschen, and Kaiser cite OECD’s approach to vulnerability (Thorn, 2009). They claim that:

the overall picture indicates a need to consider people with low numeracy as a vulnerable group, either because of prior schooling or due to extant circumstances. For the latter, the access and willingness to participate in adult education and training becomes as relevant as the exposure to family and job requirements regarding the use of numeracy skills. (Gal, Grotlüschen, & Kaiser, 2020)

In the ZDM special issue, there are some articles published so far (online first), visibilizing key examples of vulnerability. Jurdak (2019) talks about migrants as a vulnerable population. Migrant living in the host country usually face many challenges, not only in terms of language issues, but also in numeracy practices. Civil, Jablon, and Salazar (2019), in the same issue, further elaborate this idea. They provide two examples of using mathematical task with rich potential for adult numeracy education. The first case draws on a group of mothers working on addition and subtraction. The second one analyzes problem-solving. Civil has a large trajectory highlighting how Latina families deal with mathematics across the US border.

Ethnic minorities has been also included among the literature as a vulnerable population, in terms of numeracy practices (Díez-Palomar, Flecha, Garcia-Carrion, & Molina-Roldan, 2018). Díez-Palomar and his colleagues (2018) talk about Roma people who have been excluded in Europe for many centuries now. Their numeracy abilities are neglected by the school system. On the contrary, the school legitimated discourse dismisses Roma numeracy practices, although in other contexts the same practices are presented as crucial for a high-quality number sense development.

Another group that has been defined as vulnerable is what Christou and Puigvert (2011) define as other women. They are women without academic grades, who never went to school (or they dropped out at very early stages, because family and/or contextual duties as taking care of siblings, bringing a salary to home, etc.). Díez-Palomar (2020) describes a successful educational action (dialogic mathematics gatherings) as a resource to empower those women's numerical abilities and critical mathematical thinking. O'Campo (2020) is replicating the same type of intervention in Mexico City, with a similar success. She describes how women use dialogic mathematics gatherings to recover their motivation towards mathematics and numeracy, going back to school to reassume their learning trajectories, looking for new opportunities to improve their chances to obtain better jobs, and better living conditions.

Dias da Silva (2019) contributes to the discussion of vulnerable populations and numeracy with the case of women in poor Brazilian neighborhoods. He narrates how women participating in a popular adult school (Centro Integrado de Educación de Jóvenes y Adultos) face the challenges of our current society, in a context of lack of resources and opportunities.

Another interesting piece is the article by Grotlüschen, Buddeberg, Redmer, Ansen and Dannath (2019) on the relationship between vulnerable subgroups and numeracy practices. They notice that people living in vulnerable conditions (low income) tend to be more likely to use calculation more often than other people, without technical devices. They used to do so, in order to calculate personal budgets, for instance, which are a crucial issue for most of them. This study reveals that some PIAAC results, such as the correlation between low numeracy skills and low numeracy practices must to be further analyzed, because it is not always true (for instance, when we introduce the variable "level of income.")

Finally, another big source of studies on vulnerability and numeracy is in the field of financial literacy (Lusardi, 2012; Lusardi & Mitchell, 2008, 2014). Evidence consistently suggests that higher levels of financial literacy tend to protect individuals, especially older people, towards exclusion and social impairment (James, Boyle, Bennett, & Bennett, 2012). Numeracy becomes a predictor of risk factor for elder people to be victims of financial exploitation (Wood, Liu, Hanoch, & Estevez-Cores, 2015).

Evaluation and assessment

Assessment has been one of the major cornerstones for numeracy since it became a "popular" term, in the Crowther Report (1959), and twenty years later, in the Cockcroft Report (1982). Those reports, that became international benchmarks in the field, defined numeracy as a crucial aspect of political, economic and social interest. Many governments around the World allocate resources to participate in international surveys, aiming at knowing the level of their inhabitants' numeracy.

In the Crowther Report (1959), numeracy was conceptualized as in the following quote:

Literacy has long been important, and its value is as great as ever. Just as by "literacy", in this context, we mean much more than its dictionary sense of the ability to read and write, so by "numeracy" we mean more than mere ability to manipulate the rule of three. When we say that a scientist is "illiterate", we mean that he is not well enough read to be able to communicate effectively with those who have had a literary education. When we say that a historian or a linguist is "innumerate" we mean that he cannot even begin to understand what scientists and mathematicians are talking about (...) It is perhaps possible to distinguish two different aspects of numeracy (...) On the one hand is an understanding of the scientific approach to the study of phenomena - observation, hypothesis, experiment, verification. On the other hand, there is the

need in the modern world to think quantitatively, to realise how far our problems are problems of degree even when they appear as problems of kind. Statistical ignorance and statistical fallacies are quite as widespread and quite as dangerous as the logical fallacies which come under the heading of illiteracy. The man who is innumerate is cut off from understanding some of the relatively new ways in which the human mind is now most busily at work. Numeracy has come to be an indispensable tool to the understanding and mastery of all phenomena, and not only of those in the relatively close field of the traditional natural sciences. (Crowther Report, 1959, p. 270-C271)

Twenty years later, on the Cockcroft Report (1982) numeracy was defined as:

The words 'numeracy' and 'numerate' occur in many of the written submissions which we have received. In the light of our discussion in the preceding paragraphs we believe that it is appropriate to ask whether or not an ability to cope confidently with the mathematical needs of adult life, as we have described them, should be thought to be sufficient to constitute 'numeracy' (...) We would wish the word 'numerate' to imply the possession of two attributes. The first of these is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life. The second is an ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease. Taken together, these imply that a numerate person should be expected to be able to appreciate and understand some of the ways in which mathematics can be used as a means of communication (Cockcroft Report, 1982, p. 11-12)

Numeracy was taken as a serious thing by Governments all around the World (currently there are over 30 countries participating in the largest international survey on numeracy, which is PIAAC). Between 1994 and 1998, the International Adult Literacy Study (IALS) was held in 22 countries and regions. IALS was the first-ever, large-scale, international comparative assessment, designed to assess literacy and numeracy. It was held in three phases (1994, 1996 and 1998).

In 2003 IALS was replaced by the Adult Literacy and Life Skills Study (ALL), developed under OECD. ALL introduced the idea of numerate behavior to be able to understand and observe the way in which people manage mathematical situations in real life. The focus in ALL was on numerate behavior. It was conducted between 2003 and 2008 (two cycles). Numeracy was defined in ALL as the knowledge and skills required to manage mathematical demands of diverse situations.

In 2011 the Program for the International Assessment of Adult Competencies (PIAAC) took the place of international assessment large-scale studies. PIAAC was also developed under the auspices of the Organization for Economic Cooperation and Development (OECD). In the first cycle of PIAAC, a total of 24 countries participated (in 2012), nine more in 2014, and five more in 2017. Now, PIAAC Cycle 2 is running (2018-2023), involving 33 countries from the beginning.

The goal of PIAAC is to assess and compare the basic skills and the broad range of competencies of adults around the world. The assessment focuses on cognitive and workplace skills needed for successful participation in 21st-century society and the global economy.

PIAAC maintains many of the trend items coming from previous international surveys (IALS and ALL).

In Europe, in 2009-2011 another international survey was created, within the frame of the In balance project: The European Numeracy Framework (Groenestijn, Diez-Palomar, and Kanes, 2011). That was the first attempt to create a European framework to assess and to create a common base in European countries for learning and teaching numeracy in adult education in Europe (see detailed description in Annex 1).

Each of these international surveys defined levels of complexity (or proficiency levels) in terms of literacy and numeracy.

IALS used the term quantitative literacy (instead of numeracy). The word ‘numeracy’ came into being after IALS. According to IALS, quantitative literacy is:

the knowledge and skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials, such as balancing a checkbook, calculating a tip, completing an order form, or determining the amount of interest on a loan from an advertisement.

In order to assess it, experts used the levels or proficiency summarized in Table 6.

Table 6. Levels of proficiency of quantitative literacy (IALS)

Level	Quantitative literacy
Level 1 (0 to 225)	Although no quantitative tasks used in the IALS fall below the score value of 225, experience suggests that such tasks would require the reader to perform a single, relatively simple operation (usually addition) for which either the numbers are already entered onto the given document and the operation is stipulated, or the numbers are provided and the operation does not require the reader to borrow.
Level 2 (226 to 275)	Tasks in this level typically require readers to perform a single arithmetic operation (frequently addition or subtraction) using numbers that are easily located in the text or document. The operation to be performed may be easily inferred from the wording of the question or the format of the material (for example, a bank deposit form or an order form).
Level 3 (276 to 325)	Tasks found in this level typically require the reader to perform a single operation. However, the operations become more varied—some multiplication and division tasks are found in this level. Sometimes two or more numbers are needed to solve the problem and the numbers are frequently embedded in more complex displays. While semantic relation terms such as “how many” or “calculate the difference” are often used, some of the tasks require the reader to make higher order inferences to determine the appropriate operation.
Level 4 (326 to 375)	With one exception, the tasks at this level require the reader to perform a single arithmetic operation where typically either the quantities or the operation are not easily determined. That is, for most of the tasks at this level, the question or directive does not provide a semantic relation term such as “how many” or “calculate the difference” to help the reader.
Level 5 (376 to 500)	These tasks require readers to perform multiple operations sequentially, and they must dis-embed the features of the problem from the material provided or rely on background knowledge to determine the quantities or operations needed.

Source: OECD and Statistics Canada, 1995. Literacy, Economy and Society. Results of the First International Adult Literacy Survey, page 29.

ALL moved from using the term “quantitative literacy” towards “numeracy.” Numeracy was defined as “the knowledge and skills required to effectively manage and respond to the mathematical demands of diverse situations.”

This brief definition of numeracy is much broader than the definition of Quantitative Literacy as used in the IALS. Its key concepts relate in a broad way to situation management and to a range of effective responses (not only to application of arithmetical skills). It refers to a wide range of skills and knowledge (not only to computational operations) and to a wide range of situations that present actors with mathematical information of different types (not only those involving numbers embedded in printed materials). (Statistics Canada, 2005, p. 151)

The team of experts also included a definition for problem-solving, as “goal-directed thinking and action in situations for which no routine solution procedure is available.” (Statistics Canada, 2005, p. 197) In a way, problem-solving and numeracy were somehow connected. Table 7 summarizes the proficiency levels as defined in ALL.

Table 7. Levels of proficiency of numeracy (ALL)

Level	Quantitative literacy
Level 1 (0 to 225)	Tasks in this level require the respondent to show an understanding of basic numerical ideas by completing simple tasks in concrete, familiar contexts where the mathematical content is explicit with little text. Tasks consist of simple, one-step operations such as counting, sorting dates, performing simple arithmetic operations or understanding common and simple percents such as 50%.
Level 2 (226 to 275)	Tasks in this level are fairly simple and relate to identifying and understanding basic mathematical concepts embedded in a range of familiar contexts where the mathematical content is quite explicit and visual with few distracters. Tasks tend to include one-step or two-step processes and estimations involving whole numbers, benchmark percents and fractions, interpreting simple graphical or spatial representations, and performing simple measurements.
Level 3 (276 to 325)	Tasks in this level require the respondent to demonstrate understanding of mathematical information represented in a range of different forms, such as in numbers, symbols, maps, graphs, texts, and drawings. Skills required involve number and spatial sense, knowledge of mathematical patterns and relationships and the ability to interpret proportions, data and statistics embedded in relatively simple texts where there may be distracters. Tasks commonly involve undertaking a number of processes to solve problems.
Level 4 (326 to 375)	Tasks at this level require respondents to understand a broad range of mathematical information of a more abstract nature represented in diverse ways, including in texts of increasing complexity or in unfamiliar contexts. These tasks involve undertaking multiple steps to find solutions to problems and require more complex reasoning and interpretation skills, including comprehending and working with proportions and formulas or offering explanations for answers.

Level 5 (376 to 500)	Tasks in this level require respondents to understand complex representations and abstract and formal mathematical and statistical ideas, possibly embedded in complex texts. Respondents may have to integrate multiple types of mathematical information, draw inferences, or generate mathematical justification for answers.
----------------------	--

Source: Statistics Canada and Organization for Economic Cooperation and Development, 2005. Learning a Living: First Results of the Adult Literacy and Life Skills Survey, page 17.

Nowadays we use PIAAC at the international landscape to measure the level of numeracy of adults. AS we can read in the website devoted to explaining how PIAAC works, “PIAAC measures literacy and numeracy in both paper and computer modes. Items that measure problem solving in technology-rich environments are only computer-administered.”

In the PIAAC numeracy framework, numeracy is defined as:

The ability to access, use, interpret, and communicate mathematical information and ideas, to engage in and manage mathematical demands of a range of situations in adult life. (Gal et al., 2009)

Table 8 summarizes the levels of proficiency as defined within the first Cycle of PIAAC.

Table 8. Levels of proficiency of numeracy (PIAAC)

Achievement level and score range	Task descriptions
Below Level 1	Tasks at this level require the respondents to carry out simple processes such as counting, sorting, performing basic arithmetic operations with whole numbers or money, or recognizing common spatial representations in concrete, familiar contexts where the mathematical content is explicit with little or no text or distractors.
Level 1	Tasks at this level require the respondent to carry out basic mathematical processes in common, concrete contexts where the mathematical content is explicit with little text and minimal distractors. Tasks usually require one-step or simple processes involving counting, sorting, performing basic arithmetic operations, understanding simple percents such as 50%, and locating and identifying elements of simple or common graphical or spatial representations.
Level 2	Tasks at this level require the respondent to identify and act on mathematical information and ideas embedded in a range of common contexts where the mathematical content is fairly explicit or visual with relatively few distractors. Tasks tend to require the application of two or more steps or processes involving calculation with whole numbers and common decimals, percents and fractions; simple measurement and spatial representation; estimation; and interpretation of relatively simple data and statistics in texts, tables and graphs.

Level 3	Tasks at this level require the respondent to understand mathematical information that may be less explicit, embedded in contexts that are not always familiar and represented in more complex ways. Tasks require several steps and may involve the choice of problem-solving strategies and relevant processes. Tasks tend to require the application of number sense and spatial sense; recognizing and working with mathematical relationships, patterns, and proportions expressed in verbal or numerical form; and interpretation and basic analysis of data and statistics in texts, tables and graphs.
Level 4	Tasks at this level require the respondent to understand a broad range of mathematical information that may be complex, abstract or embedded in unfamiliar contexts. These tasks involve undertaking multiple steps and choosing relevant problem-solving strategies and processes. Tasks tend to require analysis and more complex reasoning about quantities and data; statistics and chance; spatial relationships; and change, proportions and formulas. Tasks at this level may also require understanding arguments or communicating well-reasoned explanations for answers or choices.
Level 5	Tasks at this level require the respondent to understand complex representations and abstract and formal mathematical and statistical ideas, possibly embedded in complex texts. Respondents may have to integrate multiple types of mathematical information where considerable translation or interpretation is required; draw inferences; develop or work with mathematical arguments or models; and justify, evaluate and critically reflect upon solutions or choices.

Theories and frameworks

Lerman (2010) has a chapter in the book titled *Theories of Mathematics Education* edited by Bharath Sriraman and Lyn English, providing a wide overview of the theories used in mathematics education. In his chapter, Lerman (2010) discusses how mathematics education draws on Psychology, Sociology, Education, Linguistics, and even Philosophy. He is able to even provide some quantitative data about the percentage of every single tradition by looking at the papers published / presented in the main international conferences and journals in the area (PME, ESM, JRME), from the decade of the 1990s, highlighting that traditional psychological and mathematics theories are the ones more prevalent among the literature. The fact that in education (mathematics education, or ALM) there is a plurality of disciplines that overlap in the effort to explain how learning mathematics works, is not a new claim.

In this literature review, we scope this analysis based on disciplines (Psychology, Sociology, Mathematics, etc.), to focus on a more specific approach, based on an epistemological view of the theoretical frameworks. In this sense, we can claim that ALM is a very eclectic field. There is a plethora of different theoretical approaches in which researchers may draw to frame their studies. Constructivism, Radical Constructivism, CHAT Theory, Enactivism, Social Constructivism, Realistic Mathematics, Embodied Cognition, Women Studies, Grounded Theory, Ethnomathematics, Semiotics, Dialogic Learning, Freirean approaches, Structuralism and post-Structuralism, Bricolage, ... The list of theoretical approaches is huge. Almost in every single paper we can find references to one or another approach. The literature review reveals that there is a vast diversity in terms of frameworks in ALM. However, among all of them, it is possible to summarize them in several theories that are the most prevalent throughout the literature (see Table 9).

Table 9. Main theoretical approaches about ALM and numeracy

Sociocultural approach	((Bennison, 2015, 2016; Coben, 2006a; Coben & Alkema, 2017; Evans, 2006; FitzSimons & Mlcek, 2004; Ní Ríordáin, Coben, & Miller-Reilly, 2015; Nicholas, Fletcher, & Davis, 2012; Wedege, 2004; Wedege & Evans, 2006; Yasukawa & Johnston, 2001)
Paulo Freire and Freirean studies	(Coben, 2000, 2013; Frankenstein, 2001)
Realistic Mathematics	(Coben et al., 2008; Gibney, 2014; Ginsburg, Manly, & Schmitt, 2006; Hoogland, Bakker, Koning, & Gravemeijer, 2012; Jablonka, 2015; Oughton, 2009)
Dialogic Learning	(Civil, 2001; Civil & Bernier, 2006; Díez-Palomar, Gimenez, & Garcia Wehrle, 2005; Díez-Palomar & Roldan, 2009; Díez-Palomar, 2015; Díez-Palomar, Menéndez, & Civil, 2011; Díez-Palomar, Gimenez, & Wehrle, 2006)
Critical Studies	(Evans et al., 2019, 2012; Galligan, 2013; Geiger, Forgasz, et al., 2015; Geiger, Goos, & Forgasz, 2015; B. Johnston, 1994; Jurdak, 2016; Kemp, 2005; Lake, 2002; Stoessiger, 2002; Strough, Bruin, & Peters, 2015; Yasukawa & Black, 2016; Yasukawa et al., 2018, Benn, 1997)
Women studies and gender	(Almenberg & Dreber, 2015; Díez-Palomar, 2020; Driva, Lührmann, & Winter, 2016; Henningsen, 2002)

Among the literature, the most frequent theoretical frame are the so-called socio-cultural theories. References to Vygotsky and his followers (Bruner, Wertsch, Luria, Valsiner, Cole, Moll, etc.) are vast, probably because adults learning mathematics is conceptualized as a situated practice, in-context, culturally and socially determined. For the same reason, references to Piaget and his work are almost nonexistent, probably because Piaget argued that learning comes in stages, hence adulthood is far away from that conceptualization. In fact, the literature on ALM, and numeracy, largely demonstrates that adults are able to learn at any age, no matter the content.

According to Coben (2006a), socio-cultural theories in mathematics education (Atweh, Forgasz, & Nebres, 2001) were adopted in the field of ALM in the late 1990s and early 2000s. She mentions the work of Lave (1988, Lave & Wenger, 1991) who reported the mathematics embedded in situated practices, as shopping in a supermarket. Those types of situations were clearly connected (and familiar) with ALM debates. The same happened with Teresinha Nunes and her work in Brazil (Carraher, 1991; Carraher, Carraher, & Schliemann, 1988). The situations narrated in her amazing research with the *meninos da rua* inspired many ALM researchers to look onto everyday practices from the lenses of numeracy, numerate behavior, and numerical practices. This is the case of Evans (2000), for instance. According to Coben (2006a),

What these approaches have in common is a deep regard for the social context in which people's actions, including their mathematical actions and interpretations of information involving mathematics, have meaning. They are based in a view of the human being as a social being and of mathematics as human activity. (p. 100)

Numeracy (within this theories) is defined as a social practice. This fact opens the floor to many potential studies. However, it also provokes some questions. Fundamental debates around numeracy as a social practice include whether numeracy is unique or it refers to a plurality of “numeracies” (since numeracy is embedded in context, then, depends on the type of context, hence, there is more than one “numeracy”); what happens when “numeracy” is so embedded in social practices or situations that it becomes “invisible” in individuals’ social practices. Coben (2006a) asks: “should researchers -or teachers- avoid imputing mathematical thinking and other activity to individuals in situations where those individuals deny that mathematics is involved?” (p. 103)

Freire is another decisive reference in the field, because he was able to alphabetize thousands of peasants (aparceros or trabalhadores rurais sem terra) in poor areas in Brazil. He demonstrated with massive evidence that drawing on the individual’s context, as an educational strategy to sense to the curriculum, transformed education of the oppressed people in a practice of freedom. Many ALM studies have rooted in this approach, since most of the times adults (in the field of ALM) are vulnerable populations going back to the school for a second chance. A remarkable example of Freire’s influence in the field of ALM, in terms of numeracy, is Frankenstein (2001) work. She uses examples from activism situations (riots, unemployment, distribution of the wealth, etc.) to put adults in front of situations in which they must use their numeracy skills to provide a critical answer. Actions such as examining critically data on people unemployed in US, including rates, for instance, allowed her to bring numeracy to teaching, as a critical instrument. Drawing on this approach, studies become social, political, and critical. Another example of this approach is Coben’s (2013) book on *Radical Heroes. Gramsci, Freire and the Politics of Adult Education*, in which she discusses the contributions of these two heroes within the context of practical forms of resistance, as the one reported by Gelsa Knijnik (1996) with the landless people’s movement in Brazil. Numeracy again, is an instrument for critical thinking and action.

Different is the case of Realistic mathematics education. This approach, rooted in Netherlands and in Freudenthal’ work, use realistic situations to develop mathematics. Students are asked to move from informal situations to formal representations. There are a huge number of studies under this framework (Dickinson & Hough, 2012; Gravemeijer, Stephan, Julie, Lin, & Ohtani, 2017; Treffers, 1993; Van den Heuvel-Panhuizen & Drijvers, 2014; Webb, Van der Kooij, & Geist, 2011), thus is impossible to cite all of them (or even a part of them). However, probably one of the most important aspects for ALM and numeracy coming from this theoretical approach is the relevance of “real” contexts, to use them as generative situations to ask adults to develop their numeracy skills. This is what we can find in Gibney, (2014), Hoogland et al. (2012) and Oughton (2009), for example. Hoogland and his colleagues (2012), for instance, demonstrated with primary, secondary, but also with VET students, that using image-rich problems in a series of 21 numeracy problems, increased significantly students’ abilities in solving quantitative problems from daily life.

Another theoretical framework that has entered mathematics education is dialogic learning (Flecha, 2000). There is a special issue published in ZDM dedicated to this approach (Bakker, Smit, & Wegerif, 2015). In the field of ALM, and numeracy, there are many studies adopting this framework, since the early 2000s (Civil, 2001; Civil & Bernier, 2006; Díez-Palomar, Gimenez, & Garcia Wehrle, 2005; Díez-Palomar & Roldan, 2009; Díez-Palomar, 2015; Díez-Palomar, Menéndez, & Civil, 2011; Díez-Palomar, Gimenez, & Wehrle, 2006). The cornerstone of dialogic learning is the assumption that learning comes as a consequence of egalitarian dialogue between individuals who, drawing on validity claims, are able to justify their claims. When they do so, they must use some type of argument or explanation, that has to be verified (shared, understood) by all the participants in the dialogic event (Díez-Palomar & Cabré, 2015, Díez-Palomar, Chan, & Clarke, in press). There are examples of how adults (especially women) draw on their own daily life experiences to make meaning to numeracy contents (Díez-Palomar, 2020).

Although we present here Critical Studies as a separated theoretical framework (because there is a tradition to do so), in fact, it is impossible to separate this approach from almost all the theoretical frameworks presented above. Critical studies are transversal and highly frequent among research in numeracy and ALM (Jurdak, 2016). Critical studies assume that individuals can (and must) think critically, taking action based in their (our) own informed judgement. In order this judgement to be informed, at least in the field of ALM and numeracy, adults have to be aware of their (our) own numeracy skills (sometimes invisibilized, as many studies have proved). Research in ALM and numeracy, under this framework, tend to make this knowledge explicit, giving back to adults the power and capacity of take informed action. The main international assessment surveys (PIAAC, etc.) have promoted this critical thinking among many of their items, designed to measure numeracy among adults.

Finally, another big category of theories in ALM and numeracy is Women Studies, probably because most of the times, when we talk about numeracy in ALM, it refers to women. As critical thinking, women studies is a transversal framework, that we can encounter in many different traditions (Butler, 2004; Flecha, Puigvert, & Rios, 2013; Puigvert, 2014; Puigvert, Gelsthorpe, Soler-Gallart, & Flecha, 2019; Shor, 1987; Talburt & Steinberg, 2000). Studies in ALM and numeracy, drawing on this approach, highlight the big potential of numeracy for women to liberate themselves from oppression and social exclusion (Dias da Silva, 2019; O'Campo 2020). Research brings us examples of case studies, where the situation of women and groups of women are reported, to notice how they are able to overcome inequalities increasing their numeracy skills (or making them visible).

As we have told at the beginning of this section, there are more theoretical approaches, and it is almost impossible to mention all of them. However, we have reported the main ones in this review.

Contexts

There is a huge range of different contexts in which numeracy has been explored (see Table 10). We have examples of researches investigating numeracy practices at workplace environments (laboratory workers, fabrics, chemistry, automotive sector, forest guards, etc.), in banking situations (transactions, etc.), in the supermarket, shopping practices, a huge amount of studies related to diverse professions, like nursing practices, construction, carpentry, kiwifruit orchards, managing invoices, numeracy to become a critical citizen (reading graphs, understanding rates in an election procedure, etc.), commercials, using ICT, numeracy at home-practices, etc.

Table 10. Main contexts emerging from the literature on ALM and numeracy

Workplace, VET, etc.	(Boistrup, 2014; L. Boistrup & Keogh, 2017; Coben et al., 2008; Coben & Weeks, 2014; CREA, 1993; Dalby & Noyes, 2015; Díez-Palomar, 2019; Evans, Yasukawa, Mallows, & Creese, 2017; FitzSimons, 2001b, 2004a, 2005, 2013, 2014; FitzSimons & Mlcek, 2004; Hoogland, 2006; Hoyles, Noss, & Pozzi, 2001; Hutton et al., 2010; Johansson, 2014; Johnson, 2007; Kelly, 2011, 2019; Kent, Noss, Guile, Hoyles, & Bakker, 2007; Keogh, Maguire, & O'Donoghue, 2014, 2019a, 2019b, 2019c, 2019d; Keogh, Maguire, & O'Donoghue, 2012; Marks, Hodgen, Coben, & Bretscher, 2016; Marr & Hagston, 2007; Nicholas et al., 2012; Noss & Hoyles, 1996; O'Donoghue & Maguire, 2005; Safford-Ramus, 2017; Safford-Ramus, Misra, & Maguire, 2016; Straesser, 2000; O'Donoghue, & Maguire, 2005; Yasukawa et al., 2018)
ICT	(FitzSimons, 2001a; Goodman, Finnegan, Mohadjer, Krenzke, & Hogan, 2013; Hoogland & Tout, 2018; Hu, Daley, & Warman, 2019; Mellar, Kambouri, Sanderson, & Pavlou, 2004; Triantafillou & Potari, 2006; van der Wal, Bakker, & Drijvers, 2017; Williams, 2003; Xiao, 2016; Nicol & Anderson, 2000)
Home, community, shopping, etc.	(Baker, 1998; Baker, Street, & Tomlin, 2000; Baker et al., 2003; Coben & Alkema, 2017; Lave, 1988; Sterling et al., 2018; Street et al., 2005; Swain et al., 2005; Tomlin, Baker, & Street, 2002)
Parents	(Allestaht-Snider, 2006; Ben-Joseph, Dowshen, & Izenberg, 2009; Civil, 2001; Civil & Bernier, 2006; Civil, Díez-Palomar, Menéndez, & Acosta-Iruiqui, 2008; Díez-Palomar, 2011; Díez-Palomar et al., 2011; Ginsburg & Gal, 1996; Larrotta & Serrano, 2011; Niklas, Cöhrssen, & Tayler, 2016; Tomlin et al., 2002; Tout & Schmitt, 2002)
Social movements, participation, policy, citizenship, etc.	(Almenberg & Dreber, 2015; Civil & Bernier, 2006; Díez-Palomar, 2015a; Díez-Palomar et al., 2018; Coben, 2006a; Evans et al., 2019; Gal, 2000; Windisch, 2016; Erickson, 2016; Oughton, 2009; Johnston, 1999; FitzSimons & Coben, 2009; Thorstad, 1992)
Assessment	(Díez-Palomar, Hoogland, Geiger, 2019; Gal et al., 2009, 2005; Gal & Tout, 2014; Goodman et al., 2013; Hoogland, Auer, Díez-Palomar, O'Meara, Van Groenestijn, 2019; Hoogland, Díez-Palomar, Maguire, 2018; Hoogland & Tout, 2018; Hutton et al., 2010; Mercer, Warwick, & Ahmed, 2017; Tout et al., 2017; Tout & Gal, 2015; Van Groenestijn, 2002; Weeks, Hutton, Coben, Clochesy, & Pontin, 2013)

Since numeracy is embedded in informal practices, the number of contexts where we may find numeracy practices (and a particular case study looking at them) may be out of count. This is due to the nature of numeracy, itself, as a formal, but also informal and non-formal content.

Probably, the largest context that has been studied in ALM and numeracy is workplace. There are important reviews in that field (FitzSimons, 2013), and the number of different jobs explored from the lenses of numeracy is huge (see Table). For instance, CREA (1993) in the early 1990s provided one of the unique studies on adults' numeracy at the workplace in Spain. This study recalls the numerical skills that workers in the major factory of cars (SEAT) in Spain at that time used to perform their duties in the working place. A recent study in the same field (workplace) is the one published by John J. Keogh, Theresa Maguire and John O'Donoghue (2019). Drawing on Keogh's dissertation, they analyze the (invisible) mathematical skills embedded in different types of work (a warehouse picker, a van driver, a customer servant, and a warehouse operative). Laia Saló also presents a great analysis of the numeracy skills associated to a farmer and a cabinetmaker, in her PhD dissertation (2020).

In the last years, some recent researchers are paying more and more attention to the impact of ICT within the realm of ALM and numeracy (Hoogland, Auer, Díez-Palomar, O'Meara, Van Groenestijn, 2019; Hoogland & Tout, 2018). This is probably one of the contexts that would increase more in the near future, especially because the rise of the big data and the new opportunities for that big data to be integrated in everyday practices, as well as at workplace environments (5G, IoT, etc.). Traditional activities will be redefined in this new ICT environment (as e-shopping, e-banking, e-healthcare, etc.), thus adults will be confronted with newly situations in which they may have to put their numeracy skills into action. It is more likely that numeracy, also will be re-defined in those new terms.

Professional development

According to the literature, professional development in the field of ALM and numeracy is one of the major features that needs to be addressed in the near future. There are few works on that theme, probably because teaching training in ALM is not regulated in many countries, around the World. According to Ginsburg (2017),

A national survey in 1994 found that while more than 80% of adult students receive some math instruction, less than 5% of adult education teachers are certified to teach math (Gal & Schuh, 1994). While this survey was completed more than 20 years ago, I would doubt the findings would be much different today. (Ginsburg, 2017, p. 58)

This finding is consistent with previous studies (Tout & Schmitt, 2002). In 1994 Mullinix (1995) conducted a survey with 141 ABE math teachers. She found that 36% ended as adult's math teachers "by accident", or as "part of the program package" (24%). More than 55% of them did not have any specific training to teach mathematics and numeracy to adults. Tout and Schmitt (2002) also report similar findings in another study (Ward, 2000).

Probably, the most consistent work on teacher training and ALM is the one done by the ABE program (Massachusetts Department of Education, 2005).

Bennison (2016) claims that there are two main strategies to promote numeracy learning in schools through teacher professional development: "interdisciplinary enquiry where two or more subjects are combined to varying degrees and exploiting numeracy learning opportunities in subjects across the curriculum (Geiger et al., 2015)." (Bennison, 2016, p. 492) Drawing on a larger study titled 'the Numeracy Project', Bennison uses an adaptation of Valsiner's zone theory to claim that numeracy can be included transversally within the curriculum. He discusses the case of Michelle, a teacher teaching history who developed a lesson plan including different tasks requiring numeracy skills to be solved.

Michelle expressed the belief that numeracy exists beyond school but spoke of numeracy as the application of mathematics in other subjects and beyond school. This indicates a personal conception of numeracy that was focused on mathematical knowledge and context – only two of the five dimensions in Goos et al.'s (2014) numeracy model. Although Michelle stated that “maths could be applied in other subjects”, she gave no clear indication that she saw numeracy as an important aspect of developing historical understanding as advocated by Phillips (2002). (Bennison, 2016, p. 499)

This example suggests that many times, including numeracy within the lesson plan, or as a part of the teacher’ practice, demands an active role from the teacher’ side. Michele was very committed to the importance that numeracy has (beyond the school), hence this was crucial in their decision-making.

According to Ginsburg (2017), adult education programs are expected to develop partnerships with workplaces and industry. This is one of the main demands for ALM programs.

Findings consistently show that mathematical activity is deeply embedded within the work and is often practiced using procedures that are idiosyncratic to the workplace and that are often learned informally from coworkers. (Ginsburg, 2017, p. 59)

But is not the only one. Courses including ALM contents also must meet the national standards established by the curriculum, in each country. This puts some pressure on ALM teachers, as many of them have no the formal training to face this challenge successfully (Ginsburg, 2017; Tout & Schmitt, 2002). As Ginsburg claims (2017),

To me, these two simultaneous emphases of implementing the CCRS and preparation for the local workforce present a powerful challenge for adult math/numeracy instructors. On the one hand, the instructors are expected to teach all mathematical content areas (numbers and operations, algebra, geometry and data and statistics), emphasizing meaning and understanding. On the other hand, instruction must also prepare learners for the numeracy demands of particular workplaces. (Ginsburg, 2017, p. 59-60)

Ginsburg (2017) also acknowledges the fact that adults may have their own personal priorities when return to study. Sometimes they want to be able to “master content that they were unable to master at an earlier time in their lives”, or to support their children at home, or just to prove them that they are able to do mathematics. This also poses challenges for teachers in ALM programs, since the needs of those adults are different from the needs of adults looking for opportunities to get a job, for instance.

Drawing on all these evidences, it seems clear that professional development in ALM and numeracy practices would need to be one of the major areas of development in the near future, in the field of ALM.

Institutional support

Since the 1950s, numeracy has been considered as a great value for nations, around the World. Examples are the Crowther Report (1959) and the Cockcroft Report (1982), that have created the standards which later have been following internationally. Nevertheless, adult numeracy although being part of the principal policy programs around the World (i.e. Lifelong Learning policies, such as APEL in Europe), it is still despised in many ways (Coben et al., 2003; FitzSimons & Coben, 2009), especially when is time to create solid educational structures around numeracy and ALM. It has been usual that numeracy has been defined within second chance programs. A way to revert that trend are VET programs, but usually are aligned to a conceptualization of numeracy as associated to workplaces and particular types of employments and working skills.

As Yasukawa & Black (2016) claim:

Current dominant discourses of adult literacy and numeracy in many OECD countries foreground the economic interests of industry and nations and the benefits to their competitiveness arising from a literate and numerate workforce often at the expense of the interests of the workers themselves, and other actors in the field of adult literacy and numeracy (Hull, 1997; Jackson & Slade, 2008; Yasukawa, Brown, & Black, 2014). Thus, literacy and numeracy are now perceived primarily in terms of human capital, variously expressed as 'core', 'foundation', 'essential' or 'functional' skills that enable individuals, enterprises and nations to become more productive and competitive in the globalised economy. (Yasukawa & Black, 2016, p. ix)

In the international landscape, surveys as PIAAC now, and its precedents (ALL, IALS), demonstrate that numeracy is something serious for governments, who invest reasonable amounts of money to participate in the surveys in order to draw a picture of the state of numeracy in their countries (Belzer, 2017; Coben, Miller-Reilly, Satherley, & Earle, 2016; Evans, 2016; Gal et al., 2009; Gal & Tout, 2014; Golsteyn, Vermeulen, & de Wolf, 2016; Gustafsson, 2016; Hampf & Woessmann, 2017; Hinz, 2018; Hoogland, Díez-Palomar, Maguire, 2018; Kirsch & Lennon, 2017; Paccagnella, 2016; Rammstedt, Martin, Zabal, Carstensen, & Schupp, 2017; Renbarger, Rivera, & Sulak, 2019; Straesser, 2000; Tout et al., 2017).

Governments care from citizens' ability to deal with the globalization demands. However, as Yasukawa and Black (2016) claim,

Thus, alternative discourses about literacy and numeracy that go beyond economic interests emerge when research can attend to contemporary accounts of people's local, everyday lived experiences and what literacy and numeracy means to them. (Yasukawa & Black, 2016, p. ix)

It seems that critical discourses, socio-cultural research, situated cases studies, emerge from the grassroots, not from the policy interests. However, it is also clear, from the literature, that those studies fully inform international standardized programs (international assessment), as well as global, regional or local policy actions (UN Millennium goals, Sustainable Development Goals, Horizon2020, 7Th Framework Programme, etc.). Probably, the challenge for the future is how to merge global, international and local, contextualized landscapes.

Challenges of numeracy

Drawing on the literature review, there is a number of challenges that emerge. We group them in five categories, which are not exhaustive, and we expect to expand this list in the future, as well as to fulfill the current challenges encountered along the literature. The list of challenges includes:

- Numeracy as a social process
- The impact of ICTs on numeracy
- Critical citizenship
- Teacher training programs
- Assessment

Numeracy as a social process

Numeracy has evolved from being associated to basic arithmetic skills, towards numeracy as a social process. In the late 1990s, numeracy was the ability to make appropriate responses to a wide range of personal, institutional or societal needs (Been, 1997). The ALL numeracy expert team created the term “numerate behavior” (Groenestijn & Schmitt, 2000). Both are members of the ALL expert group (Gal et al., 1999). FitzSimons used Bernstein’s theory to define numeracy as horizontal discourse embedded in workplace, everyday activities involving the use and re-construction of mathematical knowledge (FitzSimons, 2007). Nowadays, it is widely accepted that numeracy is “the ability to access, use, interpret and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life.” (PIAAC expert group, 2009, p. 21) In the near future, numeracy is going to be more and more conceptualized as a skill embedded in social processes, which will raise the challenge for our community of practice to upload the definition to fit the future demands of the society.

The impact of ICTs on numeracy

The rise of big data and 5G networks will transform our lives. At the same time, being able to use those technologies, will demand more numerical skills, than the “quantitative literacy” that traditionally has been allocated under the concept “numeracy.” The contents of numeracy will need to incorporate aspects such as handling with probability, uncertainty, use of statistics, big data, representation of quantitative information, etc. Including ICTs within the definition of numeracy will not only mean to ask adults using new devices, but also to ask them to be able to deal with those fields of mathematics that usually have not been part of the numeracy practices. This discussion has already happened within the Numeracy Expert Group, in charge of developing the numeracy framework for the next cycle of PIAAC (2018-2023). There are other relevant works in this challenge (Evans et al., 2019; Hoogland & Tout, 2018). This is another of the big challenges that our scientific community will need to address, in the near future.

Critical citizenship

Connected to the previous challenge, we will also need also to expand our work including the item of “critical citizenship.” Being able to use quantitative information is the more and more a requirement to fully participate in our societies (i.e. voting, assemblies, participating in the meetings of the neighborhood, etc.). Community participation involves being able to understand rates, percentages, critical information to make decisions, etc. This is another challenge for us as a community, how to support adults to develop and use those skills in their communities.

Teacher training programs

As we have seen in the literature review, a huge percentage of teachers in ALM do not have the training to teach numeracy to adults. Although numeracy is part in almost all the curriculum standards in all nations around the World, professional training for teachers who want to work with adults is rare, as Ginsburg (2017) points out. There are few programs preparing teachers to work with adults, in the field of ALM and numeracy. This is a twofold challenge: we need to develop teacher training programs around the idea of numeracy; but we also need to work with policy-makers, decisionmakers, and other stakeholders, to promote the creation of such professional programs, at the institutional level. This is a huge (and urgent) challenge for all of us.

Assessment

As a consequence of the previous challenges, our assessment programs must to be updated, to cover the new challenges and features of ALM and numeracy. The assessment tools were already developed in the previous ENF project, as paper-based and also as digital for adult education tools (Ruepert, 2011; Groenestijn, Díez-Palomar and Kanés, 2011). But due to the rise of the new ICTs (big data, social networks, etc.), previous assessment tools need to be reviewed and updated. This is something that is already happening but remains as a challenge for the future.

References

Alkema, A. (2019). The Workplace as a Context for Adult Literacy and Numeracy Learning. *Journal of Adolescent & Adult Literacy*, 63(1), 102–105.

Alleksaht-Snider, M. (2006). Urban parents' perspectives on Children's mathematics learning and issues of equity in mathematics education. *Mathematical Thinking and Learning*, 8(3), 187–195.

Almenberg, J., & Dreber, A. (2015). Gender, stock market participation and financial literacy. *Economics Letters*, 137, 140–142.

Andrews, A., & Brown, J. (2015). The effects of math anxiety. *Education*, 135(3), 362–370.

Baker, D. (1998). Numeracy as social practice. *Literacy and Numeracy Studies*, 8(1), 37.

Baker, D. (2008). Using sand to count their number: developing teachers' cultural and social sensitivities. In J. Kalman & B. Street (Eds.), *Preguntas sin respuestas. Lectura, escritura y matemáticas como prácticas sociales*. Siglo XXI. Retrieved from http://www.waalc.org.au/09conf/docs/Using_sand_to_count_Baker.pdf

Baker, D. A., Street, B. V., & Tomlin, A. (2000). Schooled and Community Numeracies: Understanding Social Factors and "Under-Achievement" in Numeracy. Retrieved from: <https://eric.ed.gov/?id=ED477841>

Baker, D., Street, B., & Tomlin, A. (2003). Mathematics as social: understanding relationships between home and school numeracy practices. *For the Learning of Mathematics*, 23(3), 11–15.

Bakker, A., Smit, J., & Wegerif, R. (2015). Scaffolding and dialogic teaching in mathematics education: Introduction and review. *ZDM*, 47(7), 1047–1065.

Barton, D., & Hamilton, M. (2012). *Local literacies: Reading and writing in one community*. Routledge.

Belzer, A. (2017). Reflections on the PIAAC Literacy and Numeracy Frameworks. *Adult Learning*, 28(3), 118–120.

Ben-Joseph, E. P., Dowshen, S. A., & Izenberg, N. (2009). Do parents understand growth charts? A national, internet-based survey. *Pediatrics*, 124(4), 1100–1109.

Bennison, A. (2015). Supporting teachers to embed numeracy across the curriculum: A sociocultural approach. *ZDM*, 47(4), 561–573.

Bennison, A. (2016). A sociocultural approach to understanding identity as an embedder-of-numeracy: A case of numeracy and history. *European Educational Research Journal*, 15(4), 491–502.

- Bloom, D. (2010). Programs and policies to assist high school dropouts in the transition to adulthood. *The Future of Children*, 89–108.
- Boistrup, B. (2014). Construing Mathematics-Containing Activities in Adults' Workplace Competences: Analysis of Institutional and Multimodal Aspects. *Adults Learning Mathematics*, 9(1), 7–23.
- Boistrup, L., & Keogh, J. (2017). The context of workplaces as part of mathematics education in vocational studies: Institutional norms and (lack of) authenticity. In Dooley, T. & Gueudet, G. (Eds.). (2017). *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education (CERME10, February 1 – 5, 2017)*. DCU Institute of Education & ERME.
- Breen, C. (2003). Fear of mathematics in adults: Moving from insights to thoughtful enactive practice. *Literacy and Numeracy Studies*, 12(2), 65.
- Butler, J. (2004). *Undoing gender*. Routledge.
- Callingham, R., Beswick, K., & Ferme, E. (2015). An initial exploration of teachers' numeracy in the context of professional capital. *ZDM*, 47(4), 549–560.
- Carpentieri, J. D., & Frumkin, L. (2010). *Adult numeracy: a review of research*. National Research and Development Centre for Literacy and Numeracy (NRDC), Institute of Education, University of London.
- Christou, M., & Puigvert, L. (2011). The role of 'Other Women' in current educational transformations. *International Studies in Sociology of Education*, 21(1), 77–90.
- Civil, M. (2001). Parents as Learners and Teachers of Mathematics: Toward a Two-Way Dialogue. Retrieved from: <https://eric.ed.gov/?id=ED480009>
- Civil, M., & Bernier, E. (2006). Exploring images of parental participation in mathematics education: Challenges and possibilities. *Mathematical Thinking and Learning*, 8(3), 309–330.
- Civil, M., Díez-Palomar, J., Menéndez, J. M., & Acosta-Irriqui, J. (2008). Parents' Interactions with Their Children When Doing Mathematics. *Adults Learning Mathematics*, 3(n2a), 41–58.
- Coates, S., Fitzpatrick, L., McKenna, A., & Makin, A. (1995). *National Reporting System*. Melbourne: Commonwealth of Australia and the Australian National Training Authority.
- Coben, D., O'Donoghue, J., and Fitzsimons, G., (ed.), (2000) *Perspectives on Adults Learning Mathematics, Research and Practice*. Kluwer Academic Publishers.
- Coben, D. (2000). Postscript: Some thoughts on Paulo Freire's legacy for adults learning mathematics. In Coben, D., O'Donoghue, J., and Fitzsimons, G., (ed.), *Perspectives on Adults Learning Mathematics, Research and Practice* (pp. 329–341). Kluwer Academic Publishers.
- Coben, D. (2006a). Social-cultural approach to adult numeracy: Issues for policy and practice. In L. Tett, M. Hamilton, and Y. Hillier (Eds.), *Adult Literacy, Numeracy and Language: Policy, Practice and Research* (pp. 96–107). Open University Press.

- Coben, D. (2006b). What Is Specific about Research in Adult Numeracy and Mathematics Education? *Adults Learning Mathematics*, 2(1), 18–32.
- Coben, D. (2013). *Radical Heroes: Gramsci, Freire and the Politics of Adult Education*. Routledge.
- Coben, D., & Alkema, A. (2017). The case for measuring adults' numeracy practices. *Journal of Research and Practice for Adult Literacy, Secondary, and Basic Education*, 6(1), 20–32.
- Coben, D., & Chanda, N. (2000). Teaching “not less than maths, but more”: an overview of recent developments in adult numeracy teacher development in England—with a sidelong glance at Australia. In Coben, D., O'Donoghue, J., and Fitzsimons, G., (ed.), *Perspectives on Adults Learning Mathematics, Research and Practice* (pp. 307–327). Kluwer Academic Publishers.
- Coben, D., Colwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V. (2003). *Adult numeracy: Review of research and related literature*. National Research and Development Centre for adult literacy and numeracy London.
- Coben, D., Hall, C., Hutton, B. M., Rowe, D., Sabin, M., & Weeks, K. W. (2008). Numeracy for nursing: The case for a benchmark. *The Changing Face of Adults Mathematics Education: Learning from the Past, Planning for the Future*, 87.
- Coben, D., Miller-Reilly, B., Satherley, P., & Earle, D. (2016). Making the most of PIAAC: Preliminary investigation of adults' numeracy practices through secondary analysis of the PIAAC dataset. *Adults Learning Mathematics - An International Journal*, 11(2), 27–40.
- Coben, D., & Weeks, K. (2014). Meeting the mathematical demands of the safety-critical workplace: Medication dosage calculation problem-solving for nursing. *Educational Studies in Mathematics*, 86(2), 253–270.
- Condelli, L. (2006). *A Review of the Literature in Adult Numeracy: Research and Conceptual Issues*. US Department of Education.
- Cooke, A. (2017). *That First Step: Engaging with Mathematics and Developing Numeracy*. Mathematics Education Research Group of Australasia.
- Cousins, S., Brindley, J., Baker, J., & Johnston-Wilder, S. (2019). Stories of mathematical resilience: how some adult learners overcame affective barriers. *Widening Participation and Lifelong Learning*, 21(1), 46–70.
- CREA. (1993). *Numeracy at the workplace*. European Commission.
- Dalby, D. M., & Noyes, A. (2015). Connecting mathematics teaching with vocational learning. *Adults Learning Mathematics*, 10(1), 40–49.
- Dickinson, P., & Hough, S. (2012). *Using realistic mathematics education in UK classrooms*. Centre for Mathematics Education, Manchester Metropolitan University. Retrieved from: https://mei.org.uk/files/pdf/rme_impact_booklet.pdf
- Díez-Palomar, J.; Hoogland, K.; Geiger, V. (2019). Numeracy in adult education: Discussing related concepts to enrich the numeracy assessment framework. In Jankvist, U. T., Van den Heuvel-Panhuizen, M., & Veldhuis, M. (Eds.). (2019). *Proceedings of the Eleventh Congress of the European*

Society for Research in Mathematics Education (CERME11, February 6 – 10, 2019). Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

Díez-Palomar, J. (2019). Adults, mathematics and work: re-enchantment of the mathematics embedded in workplaces. *Irish Educational Studies*, 39(1), 127-129.

Díez-Palomar, J. (2011). La formación de matemáticas para las familias. Una mirada desde la etnomatemática. *Revista Latinoamericana de Etnomatemática*. Retrieved from <http://www.redalyc.org/articulo.oa?id=274019437003>

Díez-Palomar, J. (2015a). Family Math: Doing Mathematics to Increase the Democratic Participation in the Learning Process. In Gellert, U., Rodríguez, J. G., Hahn, C., & Kafoussi, S. (Eds.). (2015). *Educational Paths to Mathematics: A CIEAEM Sourcebook*. Springer.

Díez-Palomar, J. (2015b). Working with Adults: A Commentary. In Gellert, U., Rodríguez, J. G., Hahn, C., & Kafoussi, S. (Eds.). (2015). *Educational Paths to Mathematics: A CIEAEM Sourcebook*. Springer.

Díez-Palomar, J. (2017). Mathematics Dialogic Gatherings: A Way to Create New Possibilities to Learn Mathematics. *Adults Learning Mathematics- An International Journal*, 12(1), 39–48.

Díez-Palomar, J., & Cabré, J. (2015). Using dialogic talk to teach mathematics: The case of interactive groups. *ZDM*, 47(7), 1299–1312.

Díez-Palomar, J., Flecha, A., Garcia-Carrion, R., & Molina-Roldan, S. (2018). Pathways to Equitable and Sustainable Education through the Inclusion of Roma Students in Learning Mathematics. *Sustainability*, 10(7). <https://doi.org/10.3390/su10072191>

Díez-Palomar, J., Gimenez, J., & Garcia-Wehrle, P. (2006). Cognitive trajectories in response to proportional situations in adult education. *Adults Learning Mathematics- An International Journal*, 1(1), 16–45.

Díez-Palomar, J., Gimenez Rodriguez, J., & Garcia Wehrle, P. (2005). Adults' dialogic productive mathematical interactions in the classroom. In M. Horne and B. Marr (Eds.), *Connecting voices in adult mathematics and numeracy: Practitioners, researchers and learners. Proceedings of the Adults Learning Mathematics (ALM) 12th Annual International Conference jointly with the Australian Council for Adult Literacy (ACAL) and in cooperation with the Australasian Bridging Mathematics Network (BMN)* (pp. 97–103). Melbourne: ALM.

Díez-Palomar, J., Menéndez, J., & Civil, M. (2011). Learning mathematics with adult learners: drawing from parents' perspective. *Revista Latinoamericana de Investigación En Matemática Educativa*, 14(1), 71–94.

Díez-Palomar, J., & Molina, S. (2009). Family Mathematics Education: Building Dialogic Spaces for Adult Learning Mathematics. In *Proceedings of the Adults Learning Mathematics Annual Meeting*. LLU.

Driva, A., Lührmann, M., & Winter, J. (2016). Gender differences and stereotypes in financial literacy: Off to an early start. *Economics Letters*, 146, 143–146.

Erickson, A. W. (2016). Rethinking the numerate citizen: Quantitative literacy and public issues. *Numeracy*, 9(2), 4.

European Commission, Education and Culture (2008). *The European Qualifications Framework for Lifelong Learning (EQF)*. Luxembourg: Office for Official Publications of the European Communities. Retrieved from: http://ec.europa.eu/education/policies/educ/eqf/index_en.html - web page of the European Commission

European Commission, Education and Culture (2005). *Fundamentals of a Common Quality Assurance Framework (CQAF) for VET in Europe*. European Commission.

European Commission, Education and Culture (2004). *Framework for key competencies*. European Commission.

Evans, J., Ruane, S., & Southall, H. (2019). Advancing social progress through critical statistical literacy. In J. Evans, S. Ruane, and H. Southall (Eds.), *Data in Society: Challenging Statistics in an Age of Globalisation* (pp. 307-319). University of Bristol Publishers.

Evans, J., Wedege, T., & Yasukawa, K. (2012). Critical perspectives on adults' mathematics education. In Clements, M. K., Bishop, A., Keitel-Kreidt, C., Kilpatrick, J., & Leung, F. K. S. (Eds.). (2012). *Third international handbook of mathematics education* (pp. 203–242). Springer Science & Business Media.

Evans, J. (2002). *Adults' mathematical thinking and emotions: A study of numerate practice*. Routledge.

Evans, J. (2006). Affect and Emotion in Mathematical Thinking and Learning: The Turn to the Social: Sociocultural Approaches Introduction: Recent Developments in Research on Affect. In Maasz, J., & Schölglmann, W. (Eds.). (2006). *New mathematics education research and practice* (pp. 233–255). Sense Publishers.

Evans, J. (2016). What to Look for in PIAAC Results: How to Read Reports from International Surveys. In Yasukawa, K., & Black, S. (Eds.). (2016). *Beyond Economic Interests: Critical perspectives on adult literacy and numeracy in a globalised world* (pp. 41–58). Springer.

Evans, J., Yasukawa, K., Mallows, D., & Creese, B. (2017). Numeracy skills and the numerate environment: affordances and demands. *Adults Learning Mathematics: An International Journal*, 12(1), 17–26.

FitzSimons, G., & Coben, D. (2009). Adult numeracy for work and life: Curriculum and teaching implications of recent research. In Maclean, R., & Wilson, D. (Eds.). (2009). *International handbook of education for the changing world of work: Bridging academic and vocational learning* (pp. 2731–2745). Springer.

FitzSimons, G. E. (2001a). *Adult Numeracy and New Technologies of Learning*. Australia: Monash University. Retrieved from: <http://www.google.co.uk/search>.

FitzSimons, G. E. (2001b). What Counts as Mathematics in Adult and Vocational Education? Numeracy for empowerment and democracy? *Adults Learning Mathematics – An International Journal*, 2(1): 21-33.

FitzSimons, G. E. (2004a). Adult numeracy/mathematics in Australian workplaces. In L. Lindberg (Ed.), *Adults Learning Mathematics. Bilding and /or training* (pp. 79–85). Göteborg: Göteborg University - Department of Education.

- FitzSimons, G. E. (2004b). An overview of adult and lifelong mathematics education. Keynote presentation at Topic Study Group 6. In *10th International Congress on Mathematics Education ICME-10*. Retrieved from: <http://www.icme-10.dk>
- FitzSimons, G. E. (2005). Numeracy and Australian workplaces: Findings and implications. *Australian Senior Mathematics Journal*, *19*(2), 27.
- FitzSimons, G. E. (2008). Adult numeracy: Crossing borders of discourse. In V. Seabright and I. Seabright (Eds.), *Crossing borders—research, reflection & practice in adults learning mathematics. Proceedings of the 13th International Conference of Adult Learning Mathematics* (pp. 35-44). ALM.
- FitzSimons, G. E. (2013). Doing Mathematics in the Workplace: A Brief Review of Selected Literature. *Adults Learning Mathematics – an International Journal*, *8*(1), 7–19.
- FitzSimons, G. E. (2014). Commentary on vocational mathematics education: where mathematics education confronts the realities of people’s work. *Educational Studies in Mathematics*, *86*(2), 291–305.
- FitzSimons, G. E., & Godden, G. L. (2000). Review of research on adults learning mathematics. In Coben, D., O’Donoghue, J., & Fitzsimons, G. E. (Eds.). (2000). *Perspectives on adults learning mathematics: research and practice* (pp. 13–45). Springer.
- FitzSimons, G. E., & Mlcek, S. (2004). Doing, thinking, teaching, and learning numeracy on the job: an activity approach to research into chemical spraying and handling. In *Doing thinking activity learning: Proceedings of the 12th annual international conference on post-compulsory education and training* (Vol. 1, pp. 149–156).
- Flecha, R. (2000). *Sharing words: Theory and practice of dialogic learning*. Rowman & Littlefield.
- Flecha, Ramon, Puigvert, L., & Rios, O. (2013). The New Alternative Masculinities and the overcoming of gender violence. *International and Multidisciplinary Journal of Social Sciences*, *2*(1), 88–113.
- Frankenstein, M. (2001). Reading the world with math: Goals for a criticalmathematical literacy curriculum. *The Australian Association of Mathematics Teachers Inc.*, 53.
- Freire, P. (1970a). *Pedagogy of the oppressed*. Continuum.
- Freire, P. (1970b). The adult literacy process as cultural action for freedom. *Harvard Educational Review*, *40*(2), 205–225.
- Freire, P. (2000). *Pedagogy of freedom: Ethics, democracy, and civic courage*. Rowman & Littlefield Publishers.
- Gal, I. (1993). *IssGalNumerate behaviourues and Challenges in Adult Numeracy*. National Center on Adult Literacy. University of Pennsylvania.
- Gal, I. (2000). *Adult Numeracy Development: Theory, Research, Practice. Series on Literacy: Research, Policy, and Practice*. ERIC. Retrieved from: <https://eric.ed.gov/?id=ED465100>.

- Gal, I. (2002). Systemic needs in adult numeracy education. *Adult Basic Education*, 12(1), 20–33.
- Gal, I., Alatorre, S., Close, S., Evans, J., Johansen, L., Maguire, T., ... Tout, D. (2009). PIAAC Numeracy: A Conceptual Framework. *OECD Education Working Papers*, No. 35. OECD Publishing.
- Gal, I., & Tout, D. (2014). *Comparison of PIAAC and PISA frameworks for numeracy and mathematical literacy*. OECD Publishing. Retrieved from www.oecd-ilibrary.org/docserver/5jz3wl63cs6f-en.pdf?expires=1605900452&id=id&accname=guest&checksum=74EB14E7E3780935EC519A786B5F16B1
- Gal, I., Van Groenestijn, M., Manly, M., Schmitt, M. J., & Tout, D. (2005). *Adult numeracy and its assessment in the ALL survey: A conceptual framework and pilot results*. Statistics Canada. Retrieved from: https://oggiconsulting.com/wp-content/uploads/2018/05-Further-Reading/ALLS-Aug2003_Numeracy_Framework-1.pdf
- Gal, I. (2000). The numeracy Challenge. In I. Gal (Ed.), *Adult Numeracy Development. Theory, Research, Practice*. Hampton Press, Inc.
- Galligan, L. (2013). Becoming competent, confident and critically aware: Tracing academic numeracy development in nursing. *Adults Learning Mathematics: An International Journal*, 8(1), 20–30.
- Galligan, L., & Taylor, J. A. (2008). Adults returning to study mathematics. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W.T. Seah, & P. Sullivan (Eds.) *Research in Mathematics Education in Australasia* (pp. 99–118). Sense Publishers.
- Geiger, V. (2019). Using mathematics as evidence supporting critical reasoning and enquiry in primary science classrooms. *ZDM*, 51, 929-940.
- Geiger, V., Forgasz, H., & Goos, M. (2015). A critical orientation to numeracy across the curriculum. *ZDM*, 47(4), 611–624.
- Geiger, V., Goos, M., & Forgasz, H. (2015). A rich interpretation of numeracy for the 21st century: A survey of the state of the field. *ZDM*, 47(4), 531–548.
- Gibney, J. (2014). Provoking Mathematical Thinking: Experiences of Doing Realistic Mathematics Tasks with Adult Numeracy Teachers. *Adults Learning Mathematics- An International Journal*, 9(2), 97–115.
- Ginsburg, L. (2017). What’s an Adult Numeracy Teacher to Teach? Negotiating the Complexity of Adult Numeracy Instruction. *Journal of Research and Practice for Adult Literacy, Secondary, and Basic Education*, 6(1), 57.
- Ginsburg, L., & Gal, I. (1996). *Instructional Strategies for Teaching Adult Numeracy Skills*. Retrieved from: <https://eric.ed.gov/?id=ED396156>
- Ginsburg, L., Manly, M., & Schmitt, M. J. (2006). *The components of numeracy*. National Center for Study of Adult Literacy and Learning. Retrieved from [Http://www.Ncsall.Net/Fileadmin/Resources/Research/Op_numeracy.Pdf](http://www.Ncsall.Net/Fileadmin/Resources/Research/Op_numeracy.Pdf).

- Golsteyn, B. H. H., Vermeulen, S., & de Wolf, I. (2016). Teacher literacy and numeracy skills: International evidence from PIAAC and ALL. *De Economist*, 164(4), 365–389.
- Goodman, M., Finnegan, R., Mohadjer, L., Krenzke, T., & Hogan, J. (2013). *Literacy, Numeracy, and Problem Solving in Technology-Rich Environments among US Adults: Results from the Program for the International Assessment of Adult Competencies 2012. First Look. NCES 2014-008*. National Center for Education Statistics. Retrieved from: <https://eric.ed.gov/?id=ED544452>
- Goos, M., Dole, S., & Geiger, V. (2012). Numeracy across the Curriculum. *Australian Mathematics Teacher*, 68(1), 3–7.
- Gravemeijer, K., Stephan, M., Julie, C., Lin, F.-L., & Ohtani, M. (2017). What Mathematics Education May Prepare Students for the Society of the Future? *International Journal of Science and Mathematics Education*, 15(1), 105–123.
- Grotlüschen, A., Buddeberg, K., Redmer, A., Ansen, H., & Dannath, J. (2019). Vulnerable subgroups and numeracy practices: How poverty, debt, and unemployment relate to everyday numeracy practices. *Adult Education Quarterly*, 69(4), 251-270.
- Gustafsson, J.-E. (2016). Lasting effects of quality of schooling: Evidence from PISA and PIAAC. *Intelligence*, 57, 66–72.
- Gustafsson, L., Mouwiz, L., & Turner, B. R. (2004). *Adults and Mathematics: A Vital Subject*. Nationelt centrum för matematikutbildning. Göteborgs univ.
- Hampf, F., & Woessmann, L. (2017). Vocational vs. general education and employment over the life cycle: New evidence from PIAAC. *CESifo Economic Studies*, 63(3), 255–269.
- Handler, J. R. (1990). Math anxiety in adult learning. *Adult Learning*, 1(6), 20–23.
- Hart, C. (2018). *Doing a Literature Review: Releasing the Research Imagination*. Sage.
- Henningsen, I. (2002). Gender in ALM: Women and men learning mathematics. In L. Johansen & T. Wedege (Eds.), *Numeracy for Empowerment and Democracy* (pp. 223–232).
- Hinz, S. (2018). *Literacy and Numeracy Skills of US Men and Women. Data Point. NCES 2018-164*. National Center for Education Statistics.
- Hoogland, K.; Auer, M.; Díez-Palomar, J.; O’Meara, N.; Van Groenestijn, M. (2019). Initiating a common European numeracy framework. In Jankvist, U. T., Van den Heuvel-Panhuizen, M., & Veldhuis, M. (Eds.). (2019). *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11, February 6 – 10, 2019)*. Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.
- Hoogland, K.; Díez-Palomar, J.; Maguire, T. (2018). Towards a second cycle of PIAAC. Programme for the International Assessment of Adult Competencies. In B. Kelly, D. Dalby, G. Griffiths, D. Kaye and J. Stacey (Eds.), *Boundaries and Bridges: adults learning mathematics in a fractured world. Proceedings of the 25th International Conference of Adults Learning Maths – A Research Forum (ALM)*. ALM.

Hoogland, K. (2006). Mind and gesture: The numeracy of a vocational student. In M. Horne and B. Marr (Eds.), *Connecting voices in adult mathematics and numeracy: Practitioners, researchers and learners. Proceedings of the Adults Learning Mathematics (ALM) 12th Annual International Conference jointly with the Australian Council for Adult Literacy (ACAL) and in cooperation with the Australasian Bridging Mathematics Network (BMN)* (pp. 150–158). Melbourne: ALM.

Hoogland, K., Bakker, A., Koning, J. de, & Gravemeijer, K. (2012). Comparing students' results on word problems with their results on image-rich numeracy problems. Retrieved from: <https://dspace.library.uu.nl/handle/1874/272677>

Hoogland, K., & Tout, D. (2018). Computer-based assessment of mathematics into the twenty-first century: pressures and tensions. *ZDM*, *50*, 675-686.

Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education*, *32*(1), 4-27.

Hu, M., Daley, A., & Warman, C. (2019). Literacy, numeracy, technology skill, and labour market outcomes among Indigenous Peoples in Canada. *Canadian Public Policy*, *45*(1), 48–73.

Hughes-Hallett, D. (2001). Achieving numeracy: The challenge of implementation. In L. A. Steen (Ed.), *Mathematics and Democracy: The Case for Quantitative Literacy* (pp. 93–98). NCED. Retrieved from: http://www.steen-frost.org/Steen/Papers/01math_and_dem.pdf#page=113

Hutton, M., Coben, D., Hall, C., Rowe, D., Sabin, M., Weeks, K., & Woolley, N. (2010). Numeracy for nursing, report of a pilot study to compare outcomes of two practical simulation tools—An online medication dosage assessment and practical assessment in the style of objective structured clinical examination. *Nurse Education Today*, *30*(7), 608–614.

Jablonka, E. (2003). Mathematical literacy. In Bishop, A., Clements, M. K., Keitel-Kreidt, C., Kilpatrick, J., & Leung, F. K. S. (Eds.). (2003). *Second international handbook of mathematics education* (pp. 75-102). Springer Science & Business Media.

Jablonka, E. (2015). The evolution of numeracy and mathematical literacy curricula and the construction of hierarchies of numerate or mathematically literate subjects. *ZDM*, *47*(4), 599–609.

James, B. D., Boyle, P. A., Bennett, J. S., & Bennett, D. A. (2012). The impact of health and financial literacy on decision making in community-based older adults. *Gerontology*, *58*(6), 531–539.

Jameson, M. M., & Fusco, B. R. (2014). Math anxiety, math self-concept, and math self-efficacy in adult learners compared to traditional undergraduate students. *Adult Education Quarterly*, *64*(4), 306–322.

Jarvis, P. (2004). *Adult education and lifelong learning: Theory and practice*. Routledge.

Johansson, M. C. (2014). Counting or Caring: Examining a Nursing Aide's Third Eye Using Bourdieu's Concept of Habitus. *Adults Learning Mathematics- An International Journal*, *9*(1), 69–84.

- Johnson, S. (2007). Professional development of the adult numeracy teaching workforce. In T. Maguire, N. Colleran, O. Gill, J. O'Donoghue, and K. Safford-Ramus (Eds.), *The Changing Face of Adults Mathematics Education: Learning from the past, planning for the future. Proceedings of ALM-14, the 14th International Conference of Adults Learning Mathematics—A Research Forum (ALM)* (pp. 34–52). ALM.
- Johnston, B. (1994). Critical numeracy. *Fine Print*, 16(4), 32–35.
- Johnston, R. (1999). Adult learning for citizenship: towards a reconstruction of the social purpose tradition. *International Journal of Lifelong Education*, 18(3), 175–190.
- Jurdak, M. (2016). *Learning and teaching real world problem solving in school mathematics*. Springer.
- Kanes, C. (2002). Towards numeracy as a cultural historical activity system. In P. Valero, and O. Skovsmose (Eds.), *Proceedings of the Third International Mathematics Education and Society Conference* (pp. 341–350). Roskilde University.
- Karaali, G., Villafane Hernandez, E., Taylor, J. (2016). What's in a name? A critical review of definitions of quantitative literacy, numeracy, and quantitative reasoning. *Numeracy: Advancing education in quantitative literacy*, 9(1), 1-34.
- Kelly, B. (2011). From the classroom to the workplace and back again: Developing an understanding of numeracy teaching. In G. Griffiths and D. Kaye (Eds.), *Proceedings for the 16th international Conference in Adults learning Mathematics* (pp. 120–132). London South Bank University. ALM.
- Kelly, B. (2019). Motivating adults to learn mathematics in the workplace: a trade union approach. *International Journal of Lifelong Education*, 38(2), 132–147.
- Kemp, M. (2005). *Developing critical numeracy at the tertiary level*. Unpublished PhD Dissertation. Murdoch University.
- Kent, P., Noss, R., Guile, D., Hoyles, C., & Bakker, A. (2007). Characterizing the use of mathematical knowledge in boundary-crossing situations at work. *Mind, Culture, and Activity*, 14(1–2), 64–82.
- Keogh, J. J., Maguire, T., & O'Donoghue, J. (2014). A Workplace Contextualisation of Mathematics: Measuring Workplace Context Complexity. *Adults Learning Mathematics- An International Journal*, 9(1), 85–99.
- Keogh, J. J., Maguire, T., & O'Donoghue, J. (2019a). Adults Using Mathematics in the Workplace: It's Not Mathematics, It's Just Common Sense; Part of the Job. In J.J. Keogh, T. Maguire, & J. O'Donoghue, (Eds), *Adults, Mathematics and Work: From Research into Practice* (pp. 5-17). Brill Sense.
- Keogh, J. J., Maguire, T., & O'Donoghue, J. (2019b). Job Shadowing: Four Case Studies of People Engaged in Their Job. In J.J. Keogh, T. Maguire, & J. O'Donoghue, (Eds), *Adults, Mathematics and Work: From Research into Practice* (pp. 49–112). Brill Sense.
- Keogh, J. J., Maguire, T., & O'Donoghue, J. (2019c). National Survey of People at Work in Ireland. In J.J. Keogh, T. Maguire, & J. O'Donoghue, (Eds), *Adults, Mathematics and Work: From Research into Practice* (pp. 129-139). Brill Sense.

Keogh, J. J., Maguire, T., & O'Donoghue, J. (2019d). *Adults, Mathematics and Work: From Research into Practice*. Brill Sense.

Keogh, J., Maguire, T., & O'Donoghue, J. (2012). A Workplace Contextualisation of Mathematics: Visible, Distinguishable and Meaningful Mathematics in Complex Contexts. In A. Hector-Mason and D. Coben (Eds.), *Proceedings of the 19th International Conference of Adults Learning Mathematics – A research forum (ALM19)*. Auckland, New Zealand. University of Waikato, ALM.

Kirsch, I., & Lennon, M. L. (2017). PIAAC: a new design for a new era. *Large-Scale Assessments in Education*, 5(1), 11.

Klinger, Christopher M. (2006). Challenging negative attitudes, low self-efficacy beliefs, and math-anxiety in pre-tertiary adult learners. In M. Horne and B. Marr (Eds.), *Connecting voices in adult mathematics and numeracy: Practitioners, researchers and learners. Proceedings of the Adults Learning Mathematics (ALM) 12th Annual International Conference jointly with the Australian Council for Adult Literacy (ACAL) and in cooperation with the Australasian Bridging Mathematics Network (BMN)* (pp. 164–171). Melbourne. ALM.

Klinger, Christopher Martin. (2010). Behaviourism, Cognitivism, Constructivism, Or Connectivism? Tackling Mathematics Anxiety with 'isms' for a Digital Age. *Adults Learning Mathematics (ALM)*. In G. Griffiths and D. Kaye (Eds.), *Proceedings for the 16th international Conference in Adults learning Mathematics* (pp. 146-154). London South Bank University, ALM.

Knijnik, G. (1996). *Exclusão resistência: educação matemática e legitimidade cultural*. Artes Médicas.

Kuntze, S., Lerman, S., Murphy, B., Kurz-Milcke, E., Siller, H.-S., & Winbourne, P. (2011). Professional knowledge related to Big Ideas in Mathematics—an empirical study with pre-service teachers. *From a Study of Teaching Practices to Issues in Teacher Education*, 290–306.

Kus, M. (2018). Numeracy. *Brock Education Journal*, 27(2), 58-62.

Lake, D. (2002). Critical social numeracy. *The Social Studies*, 93(1), 4–10.

Larrotta, C., & Serrano, A. (2011). Adult learners' funds of knowledge: The case of an English class for parents. *Journal of Adolescent & Adult Literacy*, 55(4), 316–325.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge University Press.

Lerman, S. (2010). Theories of mathematics education: Is plurality a problem? In Sriraman B., English L. (Eds.), *Theories of Mathematics Education. Advances in Mathematics Education*. Springer, Berlin, Heidelberg. (pp. 99–109). Springer.

Lucas, B. (2019) *The impact of Critical and Creative Thinking on achievement in Literacy and Numeracy: An initial review of the evidence*. Melbourne: Victorian Curriculum and Assessment Authority.

Lusardi, A. (2012). *Numeracy, financial literacy, and financial decision-making*. National Bureau of Economic Research.

- Lusardi, A., & Mitchell, O. S. (2008). Planning and financial literacy: How do women fare? *American Economic Review*, 98(2), 413–417.
- Lusardi, A., & Mitchell, O. S. (2014). The economic importance of financial literacy: Theory and evidence. *Journal of Economic Literature*, 52(1), 5–44.
- Mac Iver, M. A., & Mac Iver, D. J. (2009). *Beyond the Indicators: An Integrated School-Level Approach to Dropout Prevention*. George Washington University Center for Equity and Excellence in Education. Retrieved from: <https://eric.ed.gov/?id=ED539776>
- Maloney, E. A., & Beilock, S. L. (2012). Math anxiety: Who has it, why it develops, and how to guard against it. *Trends in Cognitive Sciences*, 16(8), 404–406.
- Manly, M., Tout, D., Van Groenestijn, M., & Clermont, Y. (2000). What Makes One Numeracy Task More Difficult Than Another? In M. J. Schmitt, and K. Safford-Ramus (Eds.), *Proceedings of ALM7 the Seventh International Conference of Adults Learning Mathematics – A research forum*. Tufts University, ALM.
- Marks, R., Hodgen, J., Coben, D., & Bretscher, N. (2016). Nursing students' experiences of learning numeracy for professional practice. *Adults Learning Mathematics International Journal*, 11(1), 43–58.
- Marr, B., & Hagston, J. (2007). *Thinking Beyond Numbers: Learning Numeracy for the Future Workplace. An Adult Literacy National Project Report*. NCVER: Australian Government.
- Massachusetts Department of Education. (2005). *Massachusetts Adult Basic Education Curriculum Framework for English for Speakers of Other Languages (ESOL)*. Massachusetts Department of Education Adult and Community Learning Services.
- Mellar, H., Kambouri, M., Sanderson, M., & Pavlou, V. (2004). *ICT and adult literacy, numeracy and ESOL*. NRDC.
- Mercer, N., Warwick, P., & Ahmed, A. (2017). An oracy assessment toolkit: Linking research and development in the assessment of students' spoken language skills at age 11-12. *Learning and Instruction*, 48, 51-60.
- Merrifield, J. (2000). *Equipped for the Future Research Report: Building the Framework, 1993-1997*. National Institute for Literacy.
- Mezirow, J. (2000). *Learning as Transformation: Critical Perspectives on a Theory in Progress. The Jossey-Bass Higher and Adult Education Series*. Jossey-Bass Publishers.
- Morton, T., McGuire, T., & Baynham, M. (2006). *A literature review of research on teacher education in adult literacy, numeracy and ESOL*. NRDC.
- Mullinix, B. B. (1995). *Exploring what counts: mathematics instruction in adult basic education*. PhD Dissertation. University of Massachusetts.
- Ní Ríordáin, M., Coben, D., & Miller-Reilly, B. (2015). What do we know about mathematics teaching and learning of multilingual adults and why does it matter? *Adults Learning Mathematics - An International Journal*, 10(1), 8–23.

- Nicholas, K., Fletcher, J., & Davis, N. (2012). Raising numeracy and literacy to improve vocational opportunities. *Journal of Vocational Education & Training*, 64(4), 433–451.
- Nicol, M. M., & Anderson, A. (2000). Computer-assisted vs. teacher-directed teaching of numeracy in adults. *Journal of Computer Assisted Learning*, 16(3), 184–192.
- Niklas, F., Cohrssen, C., & Tayler, C. (2016). Parents supporting learning: A non-intensive intervention supporting literacy and numeracy in the home learning environment. *International Journal of Early Years Education*, 24(2), 121–142.
- Noss, R., & Hoyles, C. (1996). The visibility of meanings: Modelling the mathematics of banking. *International Journal of Computers for Mathematical Learning*, 1(1), 3–31.
- O Campo, M. (2020). *Tertulias matemáticas dialógicas para el aprendizaje de las matemáticas en mujeres adultas no escolarizadas y sin titulación académica*. PhD Dissertation. University of Barcelona.
- O’Donoghue, J. (2002). Numeracy and mathematics. *Irish Mathematical Society Bulletin*, 48, 47–55.
- O’Donoghue, J., & Maguire, T. (2005). The individual learner, employability and the workplace: a reappraisal of relationships and prophecies. *Journal of European Industrial Training*, 29(6), 436–446.
- OECD. (2013). *The survey of adult skills: Reader’s companion*. OECD Publishing.
- OECD. (2003). *The PISA 2003 Assessment Framework: Mathematics, reading, science and problem solving knowledge and skills*. OECD Publishing.
- Oughton, H. (2009). A Willing Suspension of Disbelief?” Contexts” and Recontextualisation in Adult Numeracy Classrooms. *Adults Learning Mathematics- An international Journal*, 4(1), 16–31.
- Paccagnella, M. (2016), *Literacy and Numeracy Proficiency in IALS, ALL and PIAAC*. OECD Education Working Papers, No. 142, OECD Publishing, Paris. Retrieved from: <http://dx.doi.org/10.1787/5jlpq7qglx5g-en>
- Pozzi, S., Noss, R., & Hoyles, C. (1998). Tools in practice, mathematics in use. *Educational Studies in Mathematics*, 36(2), 105–122.
- Puigvert, Lúdia. (2014). Preventive socialization of gender violence: Moving forward using the communicative methodology of research. *Qualitative Inquiry*, 20(7), 839–843.
- Puigvert, Lidia, Gelsthorpe, L., Soler-Gallart, M., & Flecha, R. (2019). Girls’ perceptions of boys with violent attitudes and behaviours, and of sexual attraction. *Palgrave Communications*, 5(1), 56.
- Rammstedt, B., Martin, S., Zabal, A., Carstensen, C., & Schupp, J. (2017). The PIAAC longitudinal study in Germany: rationale and design. *Large-Scale Assessments in Education*, 5(1), 4.
- Renbarger, R., Rivera, G., & Sulak, T. (2019). What predicts literacy, numeracy, and problem solving for incarcerated individuals? A PIAAC examination. *Journal of Offender Rehabilitation*, 58(3), 199–219.

- Roth, W.-M. (2012). Cultural-historical activity theory: Vygotsky's forgotten and suppressed legacy and its implication for mathematics education. *Mathematics Education Research Journal*, 24(1), 87–104.
- Rychen, D. S., & Salganic, L. H. (Eds) (2003). *Key competencies for a successful life and a well-functioning society*. Hogrefe & Huber.
- Safford-Ramus, K. (2017). Research in Adult Mathematics Education: How Far Have We Come, Where Might We Go Next? *Numeracy: A Critical Skill in Adult Education*, 28.
- Safford-Ramus, K., Misra, P. K., & Maguire, T. (2016). *The Troika of adult learners, lifelong learning, and mathematics*. Springer.
- Saló, L. (2020). *Problem solving and use of mathematics at work. Hanging around farmers and cabinetmakers*. PhD dissertation. University of Helsinki.
- Shor, I. (1987). *Freire for the classroom: A sourcebook for liberatory teaching*. Heinemann Educational Books.
- Southwood, S. (2011). Taking the Fear Out of Maths. *Adults Learning*, 22(6), 14–15.
- Statistics Canada (2005). Measuring Adult Literacy and Life Skills: New Frameworks for Assessment.
- Steen, L. A. (2001). Embracing numeracy. Mathematics and Democracy: *The Case for Quantitative Literacy*, 107–116.
- Stein, S. (1995). *Equipped for the Future: A Customer-Driven Vision for Adult Literacy and Lifelong Learning*. National Institute for Literacy.
- Stein, S. (2000). *Equipped for the Future Content Standards: What Adults Need to Know and Be Able to Do in the 21st Century*. National Institute for Literacy
- Stent, A. (1977). Can math anxiety be conquered? Change: *The Magazine of Higher Learning*, 9(1), 40–43.
- Sterling, M. R., Silva, A. F., Robbins, L., Dargar, S. K., Schapira, M. M., & Safford, M. M. (2018). Let's talk numbers: a qualitative study of community-dwelling US adults to understand the role of numeracy in the management of heart failure. *BMJ Open*, 8(9), e023073.
- Stoessiger, R. (2002). An introduction to critical numeracy. In W. Morony and P. Brinkworth (Eds.), *Springboards into numeracy* (pp. 47-52). AAMT.
- Straesser, R. (2000). Mathematical means and models from vocational contexts. In Bessot A., Ridgway J. (eds) *Education for Mathematics in the Workplace* (pp. 65–80). Springer.
- Street, B. V, Baker, D., & Tomlin, A. (2005). *Navigating numeracies: Home/school numeracy practices*. Springer.

- Street, B. V., Rogers, A., & Baker, D. (2006). Adult teachers as researchers: Ethnographic approaches to numeracy and literacy as social practices in South Asia. *CONVERGENCE-TORONTO*, 39(1), 31.
- Strough, J., Bruin, W. B. de, & Peters, E. (2015). New perspectives for motivating better decisions in older adults. *Frontiers in Psychology*, 6, 783.
- Strümpfer, D. J. W. (1999). Psychosocial resilience in adults. *Studia Psychologica*, 41(2).
- Swain, J. (2005). 'Beyond the daily application': motivations for adults attending numeracy classes. *Research in Post-Compulsory Education*, 10(3), 305–324.
- Swain, J., Baker, E., Holder, D., Newmarch, B., & Coben, D. (2005). *Beyond the daily application: making numeracy teaching meaningful to adult learners*. NRDC, Institute of Education, University of London.
- Talburt, S., & Steinberg, S. R. (2000). *Thinking queer: Sexuality, culture, and education*. Peter Lang.
- Tett, L., & Maclachlan, K. (2007). Adult literacy and numeracy, social capital, learner identities and self-confidence. *Studies in the Education of Adults*, 39(2), 150–167.
- Tett, L., Maclachlan, K., Hall, S., Edwards, V., & Garside, L. (2006). *Evaluation of the Scottish Adult Literacy and Numeracy (ALN) Strategy-Final Report*. Scottish Executive, Glasgow, Scotland.
- Thorstad, I. (1992). Adult Numeracy and Responsible Citizenship. *Adults Learning (England)*, 4(4), 104–105.
- Tomlin, A., Baker, D., & Street, B. (2002). Home and School Numeracy Practices: Where are the borders and overlaps. In P. Valero, and O. Skovsmose (Eds.), *Proceedings of the 3rd International MES Conference, Copenhagen, Centre for Research in Learning Mathematics* (pp. 480–488). University of Roskilde.
- Tout, D. (2000). Numeracy Up Front: Behind the International Life Skills Survey. *ARIS Resources Bulletin*, 11(1), 1–5.
- Tout, D., Coben, D., Geiger, V., Ginsburg, L., Hoogland, K., Maguire, T., ... Turner, R. (2017). *Review of the PIAAC Numeracy Assessment Framework*. ACER.
- Tout, D., & Gal, I. (2015). Perspectives on numeracy: reflections from international assessments. *ZDM*, 47(4), 691–706.
- Tout, D., & Schmitt, M. J. (2002). *The Inclusion of Numeracy in Adult Basic Education*. Office of Educational Research and Improvement. NCSALL.
- Treffers, A. (1993). Wiskobas and Freudenthal realistic mathematics education. *Educational Studies in Mathematics*, 25(1–2), 89–108.
- Triantafillou, C., & Potari, D. (2006). Mathematical activity in a technological workplace: Results from an ethnographic study. In J. Novotná, H. Moraová, M. Krátká, and Nad'a Stehlíková (Eds.), *Proceedings of the 30th conference of the international group for the psychology of mathematics education* (Vol. 3, pp. 297–304). Charles University in Prague.

Tygel, A. F., & Kirsch, R. (2016). Contributions of Paulo Freire for a Critical Data Literacy: a Popular Education Approach. *The Journal of Community Informatics*, 12(3). Retrieved from: Retrieved from <http://www.w.ci-journal.net/index.php/ciej/article/view/1296>

UNESCO (1997). International Standard Classification for education. www.unesco.org Working group on Basic skills, foreign language teaching and entrepreneurship (2003). Key competencies – Definitions, Knowledge, Skills and Attitudes

Van Groenestijn, M. and Lindenskov, L. (eds.) (2007). *Mathematics in Action, Communalities across differences. A handbook for teachers in Adult Education*. ALL Foundation, Netherlands.

Van Groenestijn, M. (2002). *A gateway to numeracy. A study of numeracy in adult basic education*. Utrecht CD-B Pres.

Van Groenestijn, M. (2003). Numeracy: a challenge for adult education. In B. L. Madison (Ed.), *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges* (pp. 229–234). Woodrow Wilson National Foundation.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 521–525). Springer.

van der Wal, N. J., Bakker, A., & Drijvers, P. (2017). Which Techno-mathematical Literacies Are Essential for Future Engineers? *International Journal of Science and Mathematics Education*, 15(1), 87–104.

Volchok, E. (2019). Social Justice, Numeracy, and Teaching Statistics at a Community College. *Numeracy*, 12(1), 6.

Ward, J. (2000). *Arkansas GED Mathematics Instruction: History Repeating Itself*. Peppercorn Books. Retrieved from: <https://files.eric.ed.gov/fulltext/ED474043.pdf>

Webb, D. C., Van der Kooij, H., & Geist, M. R. (2011). Design research in the Netherlands: Introducing logarithms using realistic mathematics education. *Journal of Mathematics Education at Teachers College*, 2(1).

Wedege, T. (2004). *Sociomathematics: Researching adults mathematics in work*. Universitätsverlag Rudolf Trauner.

Wedege, T., & Evans, J. (2006). Adults' resistance to learning in school versus adults' competences in work: The case of mathematics. *Adults Learning Mathematics—An International Journal*, 1(2), 28–43.

Wedege, T. (2010). People's Mathematics in Working Life: Why Is It Invisible? *Adults Learning Mathematics—An International Journal*, 5(1), 89–97.

Weeks, K. W., Hutton, B. M., Coben, D., Clochesy, J. M., & Pontin, D. (2013). Safety in numbers 3: authenticity, building knowledge & skills and competency development & assessment: the ABC of safe medication dosage calculation problem-solving pedagogy. *Nurse Education in Practice*, 13(2), e33–e42.

O'Donoghue, J., & Maguire, T. (2005). The individual learner, employability and the workplace. *Journal of European Industrial Training*, 29(6), 436-446.

Williams, Joel. (2003). *The Skills for Life survey: A national needs and impact survey of literacy, numeracy and ICT skills*. TSO.

Williams, Julian, & Wake, G. (2007). Black boxes in workplace mathematics. *Educational Studies in Mathematics*, 64(3), 317-343.

Windisch, H. C. (2016). How to motivate adults with low literacy and numeracy skills to engage and persist in learning: A literature review of policy interventions. *International Review of Education*, 62(3), 279-297.

Wood, S. A., Liu, P.-J., Hanoch, Y., & Estevez-Cores, S. (2015). Importance of numeracy as a risk factor for elder financial exploitation in a community sample. *Journals of Gerontology Series B: Psychological Sciences and Social Sciences*, 71(6), 978-986.

Xiao, F. (2016). Examining Problem-Solving Skills in Technology-Rich Environments as Related to Numeracy. *International Journal of Lifelong Learning*, 38(3), 327-338.

Yasukawa, K., & Black, S. (2016). *Beyond Economic Interests: Critical perspectives on adult literacy and numeracy in a globalised world*. Springer.

Yasukawa, K., & Johnston, B. (2001). Numeracy: Negotiating the world through mathematics. In B. Atweh, H. Forgasz, and B. Nebres (Eds.), *Sociocultural Research on Mathematics Education: An International Perspective* (pp. 279-294). Routledge.

Yasukawa, K., Rogers, A., Jackson, K., & Street, B. V. (2018). *Numeracy as Social Practice: Global and Local Perspectives*. Routledge.

Yasukawa, K., Widin, J., & Chodkiewicz, A. K. (2008). The benefits of adults learning numeracy. In J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference* (pp. 1-11). Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University.

Annex 1

From

The concept of numeracy, as defined in *The European Numeracy Framework in Balance* (Grundtvig project, 504006-LLP-1-2009-1-GRUNDTVIG-GMP), by Mieke van Groenestijn, Javier Diez-Palmar and Clive Kanés (2011).

(...)

The concept of numeracy

The concept of numeracy does not have a universally accepted definition, nor agreement how it differs from mathematics. Nor is this concept universally across the EU as distinct from mathematics education more broadly construed. For some educators it implies a set of simple skills involving the ability to carry out basic computations or arithmetical operations. For others adult numeracy concerns a broad perspective that involves a range of knowledge, skills and supporting processes that enable adults to manage mathematical demands in various real-life situations. In the ENF adult numeracy is viewed as the latter. This implies not only mathematical knowledge but in particular the ways in which adults are able to use their mathematical knowledge in personal, societal and work-related situations. Our view on the concept of numeracy and numeracy competencies is based on the following documents.

- 1) In the *Equipped for the Future* (EFF) initiative in the United States, the National Institute for Literacy (NIFL) in the United States of America has sponsored efforts to define critical skill areas. The EFF project started in 1993 and was completed in 2000. EFF set four broad types of purposes for learning (Stein, 1995, 1997; Merrifield, 2000): learning for access and orientation in the world, learning as voice to one's ideas and opinions, learning for independent action, solving problems and making decisions as a parent, citizen and worker, and learning as a bridge to further learning and to keep up with a rapidly changing world.
In the EFF literacy and numeracy are seen as the core elements for functioning in personal life (family), work, society and further learning. Since people's numeracy is related to and may at times depend on people's literacy skills or other life skills, the purposes served by numeracy are expected to parallel those served by adults' literacy (Stein, 2000, Gal, 2000).
- 2) In the *Australian National Reporting System* (NRS) (Coates, Fitzpatrick, McKenna & Makin, 1994-5) numeracy has been organized in four broad categories: numeracy for practical purposes, for interpreting society, for personal organization and for knowledge. The NRS provides learners and teachers with learning outcomes on five levels for each of the domains.
- 3) The Commission Expert Group Basic Skills/ Key competencies of the European Commission (2004) describes numeracy as part of mathematical literacy. (See appendix A). From the European perspective numeracy can be viewed from three broad categories: workplace perspective, broader life perspective (including personal life and citizenship) and educational perspective (further learning).
- 4) In the numeracy framework of the Adults Literacy and Life skills study (ALL) numeracy is described as "the knowledge and skills required to effectively manage and respond to the mathematical demands of diverse situations". (Gal, van Groenestijn, Manly, Schmitt & Tout, 2005, p. 151)

Numeracy in the ALL study concerns four domains: everyday life, work-related, societal or community and further learning.

Everyday life

The numeracy tasks that occur in everyday situations are often management tasks that one faces in personal and family life. Others revolve around hobbies, personal development and interests. Representative tasks are handling money and budgets, comparison shopping, personal time management, making decisions involving travel, planning holidays, mathematics involved in hobbies like quilting or wood-working, playing games of chance, understanding sport scoring and statistics, reading maps, and using measurements in home situations such as cooking or home repairs.

Work-related

At work, one is confronted with quantitative situations that often are more specialized than those seen in everyday life. In this context, people may develop good skills in managing situations that might be narrow in their application of mathematical themes. Representative tasks are completing purchase orders, totaling receipts, calculating change, managing schedules, budgets, and project resources, using spreadsheets, organizing and packing different shaped goods, completing and interpreting control charts, making and recording measurements, reading blueprints, tracking expenditures, predicting costs, and applying formulas.

Societal or community

Adults need to know about trends and processes happening in the world around them (e.g. regarding crime, health issues, wages, pollution) and may have to take part in social events or community action. This requires that adults can read and interpret quantitative information presented in the media, including statistical messages and graphs. Also, they may have to manage situations like organizing a fund-raiser, realizing the fiscal effect of community programs, or interpreting the results of a study of the latest health fad.

Further learning

It is often also important to have numeracy skills that enable a person to participate in further study, whether for academic purposes or as part of vocational training. In either case, it is important to be able to know some of the more formal aspects of mathematics that involve symbols, rules, and formulas and to understand some of the conventions used to apply mathematical rules and principles.

The original ALL definition was adopted in the OECD Programme for the coming International Assessment of Adults Competencies (PIAAC). For this purpose, the definition of numeracy was changed to: "Numeracy is the ability to access, use, apply, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life." (Gal et al., 2009, p.21)

For the development of the ALL numeracy framework a set of five components were identified for the assessment of adults' numerate behaviour. These components were generally accepted by the PIAAC survey. The description of numerate behavior from the ALL Survey is used as starting point for the construction of the European Numeracy Reference Framework. This description says:

"Numerate behavior is observed when people manage a situation or solve a problem in a real context; it involves responding to information about mathematical ideas that may be represented in a range of ways; it requires the activation of a range of enabling knowledge, factors, and processes." (Gal, et al., 2005, p.152)

By choosing one element of each category, one can describe the needed numeracy knowledge and skills for individuals in every situation when mathematical action is required. For example:

“Numerate behavior involves managing a situation or solving a problem in everyday life by responding (estimation with money) to information concerning quantity and number that is represented by pictures (in advertisements in leaflets) and requires the activation of estimation and computational skills”. (Van Groenestijn, 2002, pp. 32-33)

Such descriptions enable teachers in adult education to observe and assess numerate behavior of individuals in functional situations. This can be done along the lines of the scheme below.

Figure 7. Components of numerate behavior in the ALL survey. (Gal et al., 2005, p.153)



Numeracy as a competence

The core meaning of numeracy includes that adults are able to use mathematical knowledge and skills adequately in real life situations. This can be described as a competence. The Definition and Selections of Competencies project (DeSeCo; see Rychen & Salganic, 2003) has defined the broader notion of competence as “the ability to meet individual or social demands successfully, or to carry out an activity or task”. DeSeCo (2002: pp. 8-9) conceptualizes competencies as internal mental structures, i.e. abilities, capacities or dispositions embedded in the individual:

“Each competence is built on a combination of interrelated cognitive and practical skills, knowledge (including tacit knowledge), motivation, value orientation, attitudes, emotions, and other social and behavioral components that together can be mobilised for effective action. Although cognitive skills and the knowledge base are critical elements, it is important not to restrict attention to these components of a competence, but to include other aspects such as motivation and value orientation.”

This DeSeCo definition was accepted by OECD and PISA and is also accepted in the ENF. This implies that assessment of adults’ numeracy competencies, in fact, can only be assessed in real life situations. The consequence is that in adult numeracy education programs the emphasis should be on “learning by doing” and assessment should focus more on using mathematics in real life situations.

In this sense it is noticed that the assessment in the ENF can only be additional to observing adults’ numeracy competencies in real life situations. For this the scheme in figure 1 can be helpful.

In order to become aware of the meaning of competence-based learning the partners in the In-Balance project discussed their national numeracy curricula on cognitive content and numeracy competencies.

The content of the ENF is based on common agreement across the partner countries. Their national and local curricula have offered content input for the ENF. Indeed, some countries have described national standards for mathematics/numeracy education. We present only three examples.

The British core curriculum for adult education was published in 2001. In this numeracy corresponds with the ability to:

- Understand and use mathematical information
- Calculate and manipulate mathematical information
- Interpret results and communicate mathematical information

The core-curriculum describes three mathematical domains:

- Number (whole numbers, fractions, decimals and percentages)
- Measures, shape and space (common measures, shape and space)
- Handling data (data and probability)

The core curriculum provides a level indication on three levels: Entry Level, Level 1 and Level 2. These levels are aligned to the British national qualifications’ framework. The Entry Level is further divided into three sub-levels.

In Austria standards in mathematics/numeracy have been described in competences that cover ISCED level 2 after 8 years of education (“Hauptschulabschluss” at the age of about 14 years). The competences exist of four central activity areas in mathematics, four content areas and three

complexity areas. In total 4x4x3 is 48 competences. The competencies in the Austrian standards concern:

Central activity areas:

- H1: presenting and modeling
- H2: calculating and operating
- H3: interpreting
- H4: argumenting and reasoning

Content areas:

- I1: numbers and measurement
- I2: variables, functional references, equations, presentation of relationships
- I3: geometrical shapes and bodies
- I4: statistical presentation and parameters

Complexity areas:

- K1: basic knowledge and skills
- K2: making connections
- K3: reflection

In the Netherlands standards have been set for language (reading and writing) and mathematics. The standards are indicated in three levels. These levels cover ISCED level 1 (after 6 years of primary school), ISCED level 2 (after 2 years of further general education preparing for higher education or 4 years of lower VET and ISCED level 3 (after 5 or 6 years of secondary school or VET).

For mathematics/numeracy the standards describe four knowledge domains:

1. numbers
2. proportions and proportional reasoning
3. measurement, shape and space, geometry
4. data processing.

Each domain is based on three parts:

- a. Give meaning, mathematical language, using formulas and doing computations
- b. Making connections between and intertwining of mathematical knowledge and procedures
- c. Problem solving

Each of the above domains and parts implies:

- knowing: concepts, facts, computations and applying problem solving procedures
- using: in school situations but also in functional everyday life situations
- understanding: knowing why, comprehension, explanation and strategic thinking.

The examples above show how countries deal with mathematical knowledge and competencies in (adult) education and VET. In the ENF framework we respect the standards of the individual European countries. The ENF only offers an overall content-based level classification on mathematical knowledge and skills that can be embedded in national and local numeracy programs. This may help countries to compare, improve or create their own standards on numeracy learning and teaching and to get their programs in line. The ENF offers examples for good practices in four domains and on five levels that can be used to develop numerate behavior. By this a common basis can be created for the quality of teaching and learning in adult education in European countries.

Starting points for the ENF level classification

In general, the ENF supports the competencies as described by the European Commission for mathematical literacy (knowledge, skills and attitudes; see appendix 1) and in the EQF (see appendix 3). In addition, the following starting points are specific for the ENF level classification.

1. The ENF comprises 5 ENF levels.

- ENF Level 1 and 2 lead to EQF and ISCED level 1
- ENF Level 3 and 4 lead to EQF and ISCED level 2
- ENF Level 5 is the bridging level between EQF and ISCED level 2 and 3.
- The levels 2 and 4 are crucial for the assessment of EQF and ISCED levels 1 and 2.

2. Each ENF level is divided into four numeracy related knowledge domains :

- Number and operations
- Proportional reasoning (proportion, ratio, decimals, fractions, percent)
- Measurement (including metric system, money, time and dimension)
- Information / Data handling (data, chance, change, simple statistics, tables, graphs, including application of knowledge and skills in the first three domains) (also including time, calendar and temperature)

3. Each ENF level refers to all facets of numerate behaviour as indicated in Figure 1 (see above).

4. Each level includes 21 boxes.

- Each item in each box represents a particular category of mathematics. This pattern will repeat at each level. This makes it possible to assess as well as horizontally per level as well as vertically through different levels, by which a profile of an individual person may occur.
- Each level has 21 boxes with at least one item in each box.

In order to be able to assess individual's numeracy competencies the ENF offer a web-based assessment tool. The items in this tool cover the EQF and ISCED levels 1 and 2.

Each item in each box on these levels meets the following criteria:

- Each item represents a context derived from a real-life situation (work, personal life, society and for further learning)
- Each item requires activation of mathematical knowledge and skills, literacy skills and problem-solving skills
- Each item has been described in the way of numerate behavior
- Each item concerns only one task
- Each task is an open-ended task, as is in real life situations
- Each task is clear and measurable with correct or not-correct.

In general: people are free to use a calculator or other means like money, a measure tape or scales, as is in real life situations.

Level 2 or level 4 is passed when a person has at least 80% correct answers per level That means 17 items correct (rounded up).

The ENF levels compared to the EQF and ISCED levels

The ENF level classification pretends to cover the EQF and ISCED levels 1 and 2. A brief overview of the ISCED and EQF levels is provided in the appendices 2 and 3.

Compared to the ISCED levels (1997) the ENF levels build on the following criteria:

Figure 8. Correspondence ENF and ISCED levels

ENF level	Adopted criteria		ISCED skill level (UNESCO, 1997)
1	0 to 4 years of formal schooling	Not yet at ISCED 1	
2	5-7 years of formal schooling formal	ISCED 1 but not more;	ISCED LEVEL 1: Equivalent of the level of skill obtained at the successful end of “primary education which generally begins at ages 5-7 years and lasts about 5 years.”
3	8-9 years of formal schooling	More than ISCED 1 but not yet ISCED 2	
4	10-11 years of formal schooling	ISCED 2 but not more	ISCED LEVEL 2: Equivalent of the level of skill at end of first stage of secondary school up to the age of 14 or 15 years.
5	12 years or more	More than ISCED 2	

In this sense, the ENF is based on these 5 levels of “school experience” that are connected to the individuals’ school trajectory. In this sense it may be noticed that the number of years of formal school experience is not automatically related to the level of education.

According to the EQF levels (2008) the ENF covers the levels 1 and 2 and bridges level 2 and 3.

Figure 9. EQF levels 1,2 and 3

EQF Level	Knowledge	Skills	Competence	Example
Level 1	Basic general knowledge	basic skills required to carry out simple tasks	work or study under direct supervision in a structured context	
Level 2	Basic factual knowledge of a field of work or study	basic cognitive and practical skills required to use relevant information in order to carry out tasks and to solve routine problems using simple rules and tools	work or study under supervision with some autonomy	lower secondary school (FI)
Level 3	Knowledge of facts, principles, processes and general concepts, in a field of work or study	a range of cognitive and practical skills required to accomplish tasks and solve problems by selecting and applying basic methods, tools, materials and information	take responsibility for completion of tasks in work or study; adapt own behavior to circumstances in solving problems	

In combination the ENF levels cover the following parts of the ISCED and EQF:

Figure 10. Correspondence ENF with ISCED and EQF levels.

ENF level	ISCED level (1997)	Compared to	EQF level (2008)
1			
2	1	End of primary education	1
3			
4	2A - type 1	End of lower sec ed or lower VET	2
5	2A –type 2	Streaming to VET	bridging 3

Complexity factors per level and per item

Situations in real life are often quite complex. Such complexity is determined by a number of factors:

- the type of context: familiarity with and transparency of a situation
- the type of information: the quantity and complexity of oral and paper-based information (use of numbers, text and pictures)
- the type of operation: number and complexity of mathematical operations, c.q. action and computations.

Such may determine the complexity of contexts and items in the ENF. This means that we will have to describe complexity factors per level and even per item in order to achieve a standard per level. The scheme below shows how to indicate levels for type of contexts, type of information and type of operation. The scheme is derived from the ALL Survey where it was used to develop items for the numeracy part of the survey. (Gal et al., 2000).

Figure 11. Complexity scheme

<p>Type of context (real life situation and problem transparency)</p>	<ol style="list-style-type: none"> 1. clear, simple situation, zero distractors. 2. clear, less simple situation, only one distractor. The problem is well defined. 3. a bit complex situation but well defined. One or two distractors. Some translation is required, but the problem can be solved by applying a simple (standard) procedure. 4. complex situation. More translation is required. The problem can be solved by applying more complex computations, mathematical models and simple formulas. 5. more complex situation. Considerable translation is required. The problem can be solved by application of advanced mathematical techniques, such as a combination of fractions and percent.
<p>Type of information (all tasks are open ended, as is in real life situations)</p>	<ol style="list-style-type: none"> 1. clear information in numbers, pictures, objects, measures, simple tables or graphs in clear contexts, in combination with little or no text. If there is text, then it is necessary for understanding the mathematical context. 2. clear information in numbers, pictures, objects, measures, simple tables or graphs, in combination with or embedded in some simple text. Part of the text may be distracting. (see type of context) 3. information in a combination of numbers, pictures, objects, measures, tables or graphs, embedded in more simple text. Part of the text may be distracting. (see type of context) 4. complex information embedded in a combination of numbers, pictures, objects, measures, tables or graphs, embedded in more complex text, including more distractors. (see type of context) 5. more complex information embedded in a combination of numbers, pictures, objects, measures, tables or graphs, embedded in heavy complex text, including more distractors.
<p>Type of operation</p>	<ol style="list-style-type: none"> 1. one step problems: only one type of mathematical operation is required. 2. two step problems: tasks may involve one or two types of operations. 3. three step problems: tasks may involve two or three same or different types of operation. 4. multiple steps required: tasks may involve multiple steps of operation. 5. multiple steps required: tasks may involve multiple steps of operation. Advanced mathematical techniques may be required (e.g. a combination of percent, fractions, proportions).

Concerning the real life situations, the items on the different levels focus on every day (personal) life, work, society and further learning in the way as described in figure 3.

Figure 12. Allocation of items to the levels in the ENF classification levels.

<p>Allocation of real-life situations and contexts to levels</p>	<ol style="list-style-type: none"> 1. everyday life and work 2. everyday life and work 3. everyday life, work and society 4. everyday life, work and society 5. work and focus on further learning
--	---

The level of items can be determined by applying the following counting system to each item. Each item can have a max of 5 points per category, depending on the level it is supposed to be.

Figure 13. Allocation of points to items based on complexities

	# points
Type of context	1-5
Type of information	1-5
Type of operation	1-5

An item can have, for example, 2 points at type of context, 3 points at type of information and 3 points at type of operation. That means a total of 8 points. This item will be an item on level 3. See interpretation of totals in table below.

Interpretation of totals:

Figure 14. level indication of items based on complexities

Level	Total # points
1	1-2-3
2	4-5-6
3	7-8-9
4	10-11-12
5	13-14-15

Setup of the content domains and items per level

In the level classification the ENF focuses on content in four domains and on five levels. There are four domains on five levels. Each level has the same structure. The content increases in difficulty and complexity per level as indicated in the complexity factors in the previous section. Each box in the domains Numbers, Proportional reasoning and Measurement represents a specific sub-domain. In the fourth domain (Processing information / data) all types of mathematical actions of the first three domains are integrated in the ways as indicated in the boxes.

The four domains are:

- Number (N)
- Proportional reasoning (P)
- Measurement (M)
- Processing information / data (info)

Figure 15. overview of domains in the level classification

Numbers (N)	Proportional reasoning (P)	Measurement (M)	Processing information / data (info)
Whole numbers and number system	Ratio/ proportion Use of language	Length/area/ (m, km, m ² , km ²)	Numbers / picture / text
Addition	Fractions/ proportion	volume / content (dm ³ , liters)	Measurement / numbers / text
Subtraction	Decimals / proportion	Mass (weight)	Proportional reasoning / numbers / text
Multiplication	Percent/proportion	Time	Tables / numbers / text
Division	Integrated fractions/decimals / percentages	Money	Graphs / numbers / text
		Shapes and space	

Translation and adaptation

The ENF provides guidelines for translation and adaption of ENF items in the database in different languages. This section presents a brief overview of translation and adaption criteria. More detailed guidelines are available in the handbook of the web tool.

The translation criteria include:

- A Intro
- B Purposes
- C Cultural aspects
- D Language aspects
- E Mathematical aspects
- F Critical elements

A. Intro

Translation of ENF assessment and exercise items asks for careful attention.

The ENF handbook provides general guidelines for translation of items to ensure that the items for the ENF assessment and for the ENF online tool are translated to target languages as best as possible to guarantee the quality of the items.

These specific guidelines are to ensure that the numeracy items are translated as best as possible and to guarantee the quality of the ENF items in all participating countries.

These guidelines for translation are based on the ALL guidelines for translation of the ALL numeracy items, to ensure that the numeracy items for the ENF are internationally soundly grounded in previous research and proved experiences.

B. Purposes

Overall purpose:

- Translators have a clear understanding how to translate the ENF items.

Specific purposes:

- Translators know how to make cultural differences in the numeracy items as small as possible

- Translators know how to act with language issues in the numeracy items to not increase reading levels of text in the items.
- Translators know how to act with mathematical issues in the numeracy items to not create differences in levels of difficulty or in mathematical operations in the items.

C. Cultural aspects

All cognitive ability tests meet the problem of cultural differences within and across countries. The ENF assessment items are supposed to measure mathematical skills across countries. Such mathematical skills can be: doing computations with money, percents, fractions, determine the actual length based on given information on a scale, compute the number of tiles for a terrace based on the actual length and width of the terrace and the given size of the tiles, etc..

Mathematical skills in the ENF numeracy items are embedded in contexts, derived from real life situations. The ENF project team has looked for stimuli in contexts that are usable across countries. Real life situations may differ across countries or even within countries and these determine the cultural differences. Therefore, there are always contexts that cannot be simply ‘translated’ to all countries. In such cases the stimulus in the context can be adjusted to more common real-life issues people in those countries are familiar with. However, in all cases we need to be sure that no changes are made that may influence the mathematical skills in the items.

D. Language aspects

The text of the original ENF assessment and exercise items comes from different languages from the participating countries and are first translated to (British) English. Every country is supposed to translate the items to English as ‘close’ as possible to the original text. That means: no changes in reading level, in length of text, transparency of text, complexity of text, and no changes in the type of text in contexts. Do not explain more and do not provide more information than is in the original text. The second step is translation from English to another language, which is not the original language. For this the same conditions as mentioned above are applied. If possible, compare the translation with the original text (e.g. German, English and Dutch).

Each item has three types of text:

- a) Text in the context
- b) Text in the item question
- c) Specific information or instructions, depending on the item.

E. Mathematical aspects:

Mathematical information in the items can be distinguished in:

- a) Mathematical information in the context
- b) Mathematical information in the item question
- c) Mathematical instruction in the item question

In addition there is one more important topic:

- d) Mathematical information regarding currencies and measurement in a and b

F. Critical Elements

For each item on the online tool and in the assessment, critical elements are described in order to determine the goal of the item, the mathematical knowledge domain, the complexity, the level and the real-life domain of the item. These critical elements are described at the section „complexity factors” and are crucial for translation to other languages.

The critical elements are processed in the online tool and bring each item in the right box. The critical elements include:

1. Goal of the item
2. Real life domain (everyday life, work, society and further learning)
3. Knowledge domain
4. Type of context
5. Type of information
6. Type of operation
7. Item question or instruction
8. Correct answer
9. Level indication

Validation of levels

In order to validate the levels 2 and 4 of the ENF classification system, a pilot research was done in four partner countries in the last quarter of 2010: Austria, Hungary, Netherlands and Spain.

The results of this research are currently being processed by three researchers, coming from King's College in London (UK), Hogeschool Utrecht, University of applied sciences (the Netherlands) and University Autònoma of Barcelona (Spain).

In addition a small study was done in the Netherlands on learning outcomes of the ENF levels 1 and 2 in the Netherlands related to the EQF.

This section of the ENF will be completed when the results are available.

Annex 2

From

The concept of numeracy, as defined in The European Numeracy Framework in Balance (Grundtvig project, 504006-LLP-1-2009-1-GRUNDTVIG-GMP), by Mieke van Groenestijn, Javier Diez-Palmar and Clive Kanés (2011).

(...)

Figure 16. Framework for key competencies - European Commission (2004)

FRAMEWORK FOR KEY COMPETENCES IN A KNOWLEDGE-BASED SOCIETY			
3.1. Mathematical literacy			
The competence consists of the following elements of knowledge, skills and attitudes as appropriate to the context:			
• Definition of the competence	• Knowledge	• Skills	• Attitudes
<ul style="list-style-type: none"> At the most basic level, mathematical literacy comprises the use of addition and subtraction, multiplication and division, percentages and ratios in mental and written computation for problem-solving purposes 	<ul style="list-style-type: none"> Sound knowledge and understanding of numbers and measures and the ability to use them in a variety of everyday contexts is a foundation skill that comprises the basic computation methods and an understanding of elementary forms of mathematical presentation such as graphs, formulas and statistics. 	<ul style="list-style-type: none"> Ability to apply the basic elements of mathematical literacy such as <ul style="list-style-type: none"> addition and subtraction; multiplication and division; percentages and ratios; weights and measures to approach and solve problems in everyday life, e.g.: <ul style="list-style-type: none"> managing a household budget (equating income to expenditure, planning ahead, saving); shopping (comparing prices, understanding weights and measures, value for money); travel and leisure (relating distances to travel time; comparing currencies and prices) 	<ul style="list-style-type: none"> Readiness to overcome the „fear of numbers“. Willingness to use numerical computation in order to solve problems in the course of day-to-day work and domestic life.

Basic mathematical literacy (“numeracy”) is a foundation skill for all subsequent learning in other domains of key competences.

Figure 17. Framework for key competences in a knowledge-based society

FRAMEWORK FOR KEY COMPETENCES IN A KNOWLEDGE-BASED SOCIETY			
3.1. Mathematical literacy (continued)			
The competence consists of the following elements of knowledge, skills and attitudes as appropriate to the context:			
Definition of the competence	Knowledge	Skills	Attitudes
<p><i>As mathematical competence develops further, it involves, as appropriate to the context, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs/charts) which have universal application in explaining and describing reality</i></p>	<p><i>Sound knowledge of mathematical terms and concepts, including the most relevant theorems of geometry and algebra. Knowledge and understanding of the kinds of questions that mathematics may offer an answer to.</i></p>	<ul style="list-style-type: none"> - <i>Ability to follow and assess chains of arguments, put forward by others, and to uncover the basic ideas in a given line of argument (especially a proof), etc.</i> - <i>Being able to handle mathematical symbols and formulae, to decode and interpret mathematical language and to understand its relations to natural language. Ability to communicate in, with, and about mathematics.</i> - <i>Ability to think and reason mathematically (mastering mathematical modes of thought, abstracting and generalizing where relevant to the question and modelling mathematically (i.e. analyzing and building models) by using and applying existing models to questions posed.</i> - <i>Being able to understand and utilize (decode, interpret and distinguish between) different sorts of representations of mathematical objects, phenomena and situations, choosing and switching between representations as and when appropriate.</i> - <i>Disposition towards critical thinking; ability to distinguish between different kinds of mathematical statements (between e.g. an assertion and an assumption, etc.); understanding of mathematical proofs and the scope and limitations of a given concept.</i> - <i>Ability to make use of aids and tools (including IT).</i> 	<ul style="list-style-type: none"> - <i>Respect for truth as the basis of mathematical thinking.</i> - <i>Willingness to look for reasons to support one's assertions.</i> - <i>Willingness to accept or reject the opinions of others on the basis of valid (or invalid) reasons or proofs.</i>

Mathematics, although intrinsically linked to numeracy, is of higher complexity. “Mathematical behavior” is about describing reality through constructs and processes which have universal application. It is best described as a combination of skills and attitudes. The definition emphasizes the importance of “mathematical activity” and acknowledges the “links with reality” as a current emphasis in math education.

Annex 3

From

The concept of numeracy, as defined in The European Numeracy Framework in Balance (Grundtvig project, 504006-LLP-1-2009-1-GRUNDTVIG-GMP), by Mieke van Groenestijn, Javier Diez-Palomar and Clive Kanes (2011).

(...)

Brief overview of relevant ISCED levels 1,2 and 3.

The International Standard Classification of Education (ISCED) was designed by the UNESCO in the 70s to be as an instrument to compile and present statistics of education internationally. ISCED was approved in the International Conference of Education held in Geneva in 1975. Since that moment, this classification has been updated several times. The current classification comes from 1997, when UNESCO pass that version during the 29th General Conference. ISCED has seven different levels, that range from pre-primary education until tertiary education (see Table below).

Table 11. ISCED levels (Unesco, 1997).

Level 0	Pre-primary education
Level 1	Primary education or first stage of basic education
Level 2	Lower secondary or second stage of basic education
Level 3	Upper secondary education
Level 4	Post-secondary non-tertiary education
Level 5	First stage of tertiary education
Level 6	Second <u>stage</u> of tertiary education (leading to an advanced research qualification, e.g., Ph.D.)

ISCED level 1 corresponds to primary level of education. According to the adult education perspective, this level lead to what we call “basic skills programmes”. That means:

“Literacy or basic skills programmes within or outside the school system are similar in content to programmes in primary education for those considered too old to enter elementary school are also included at this level because they require no previous formal education”. (ISCED, 1997)

ISCED level 2 leads to lower secondary level of education (for ENF: sub-category 2C and type 1 and type 2 are relevant)

This level has a range of sub-categories, including:
Sub-categories at this level

Type of subsequent education or destination

ISCED Level 2 programmes are sub-classified according to the destination for which the programmes have been designed to prepare students for:

- ISCED 2A: programmes designed to prepare students for direct access to Level 3 in a sequence which would ultimately prepare students to attend tertiary education, that is, entrance to ISCED 3A or 3B.
- ISCED 2B: programmes designed to prepare students for direct access to programmes at Level 3C.
- ISCED 2C: programmes primarily for direct access to labour market at the end of this level (sometimes referred to as “terminal” programmes).

Programme orientation

Programmes at Level 2 can also be subdivided into three categories based on the degree to which a programme is specifically oriented towards a specific class of occupations or trades and leads to a labour-market relevant qualification.

- Type 1 (general): education which is not designed explicitly to prepare participants for a specific class of occupations or trades or for entry into further vocational or technical education programmes. Less than 25% of the programme content is vocational or technical.
- Type 2 (pre-vocational or pre-technical): education which is mainly designed to introduce participants to the world of work and to prepare them for entry into further vocational or technical education programmes. Successful completion of such programmes does not lead to a labor-market relevant vocational or technical qualification. For a programme to be considered as pre-vocational or pre-technical education, at least 25% of its content has to be vocational or technical.
- Type 3 (vocational or technical): education which prepare participants for direct entry, without further training, into specific occupations. Successful completion of such programmes leads to a labor-market relevant vocational qualification.

On the other side, ISCED Level 3 leads to an upper secondary level of education. According with the original document of ISCED, ISCED 3 corresponds to the final stage of secondary education in most OECD countries. Instruction is often more organized along subject-matter lines than at ISCED Level 2 and teachers typically need to have higher level, or more subject-specific qualification than at ISCED

The entrance age to this level is typically 15 or 16 years. There are substantial differences in the typical duration of ISCED 3 programmes both across and between countries, typically ranging from 2 to 5 years of schooling. ISCED 3 may either be “terminal” (i.e. preparing the students for entry directly into working life) and/or “preparatory” (i.e. preparing students for tertiary education).

Annex 4

From

The concept of numeracy, as defined in The European Numeracy Framework in Balance (Grundtvig project, 504006-LLP-1-2009-1-GRUNDTVIG-GMP), by Mieke van Groenestijn, Javier Diez-Palomar and Clive Kanis (2011).

(...)

The European Qualifications Framework (EQF)

From Wikipedia, the free encyclopedia (n.d.) (Redirected from EQF)

The European Qualifications Framework (EQF) is a European Union initiative to create a translating facility for referencing academic degrees and other learning qualifications among EU member states. It is designed to allow national qualifications frameworks, for example the UK Qualifications and Credit Framework (QCF), to be cross referenced.

The EQF was formally adopted by the European Parliament and the Council on 23 April 2008.

The EQF is a common European reference framework which links countries' qualifications systems together, acting as a translation device to make qualifications more readable and understandable across different countries and systems in Europe. It has two principal aims: to promote citizens' mobility between countries and to facilitate their lifelong learning.

The EQF has eight reference levels. The learning outcomes on these levels are depicted in the following table.

Descriptors defining levels in the European Qualifications Framework (EQF)				
Level	Knowledge	Skills	Competence	Example
Level 1	Basic general knowledge	basic skills required to carry out simple tasks	work or study under direct supervision in a structured context	
Level 2	Basic factual knowledge of a field of work or study	basic cognitive and practical skills required to use relevant information in order to carry out tasks and to solve routine problems using simple	work or study under supervision with some autonomy	lower secondary school (FI)
Level 3	Knowledge of facts, principles, processes and general concepts, in a field of work or study	a range of cognitive and practical skills required to accomplish tasks and solve problems by selecting and applying basic methods, tools, materials and information	take responsibility for completion of tasks in work or study; adapt own behavior to circumstances in solving problems	
Level 4	Factual and theoretical knowledge in broad contexts within a field of work or study	a range of cognitive and practical skills required to generate solutions to specific problems in a field of work or study	exercise self-management within the guidelines of work or study contexts that are usually predictable, but are subject to change; supervise the routine work of others, taking some responsibility for the evaluation and improvement of work or	Abitur , vocational school
Level 5 ¹	Comprehensive, specialized, factual and theoretical knowledge within a field of work or study and an awareness of the boundaries of that knowledge	a comprehensive range of cognitive and practical skills required to develop creative solutions to abstract problems	exercise management and supervision in contexts of work or study activities where there is unpredictable change; review and develop performance of self and others	HND
Level 6 ²	Advanced knowledge of a field of work or study, involving a critical understanding of theories and principles	advanced skills, demonstrating mastery and innovation, required to solve complex and unpredictable problems in a specialized field of work or study	manage complex technical or professional activities or projects, taking responsibility for decision-making in unpredictable work or study contexts; take responsibility for managing professional development of individuals	Honours Bachelor Degree , vocational university (Fachhochschule) Bachelor

Level 7 ³	Highly specialized knowledge, some of which is at the forefront of knowledge in a field of work or study, as the basis for original thinking and/or research Critical awareness of knowledge issues in a field and at the interface between different	specialized problem-solving skills required in research and/or innovation in order to develop new knowledge and procedures and to integrate knowledge from different fields	manage and transform work or study contexts that are complex, unpredictable and require new strategic approaches; take responsibility for contributing to professional knowledge and practice and/or for reviewing the strategic performance of teams	Masters, vocational university (Fachhochschule) Masters
Level 8 ⁴	Knowledge at the most advanced frontier of a field of work or study and at the interface between fields	the most advanced and specialized skills and techniques, including synthesis and evaluation, required to solve critical problems in research and/or innovation and to extend and redefine existing knowledge or professional practice	demonstrate substantial authority, innovation, autonomy, scholarly and professional integrity and sustained commitment to the development of new ideas or processes at the forefront of work or study contexts including research	Doctorate

1. The descriptor for the higher education short cycle (within or linked to the first cycle), developed by the Joint Quality Initiative as part of the Bologna process, corresponds to the learning outcomes for EQF level 5.
2. The descriptor for the first cycle in the Framework for Qualifications of the European Higher Education Area agreed by the ministers responsible for higher education at their meeting in Bergen in May 2005 in the framework of the Bologna process corresponds to the learning outcomes for EQF level 6.
3. The descriptor for the second cycle in the Framework for Qualifications of the European Higher Education Area agreed by the ministers responsible for higher education at their meeting in Bergen in May 2005 in the framework of the Bologna process corresponds to the learning outcomes for EQF level 7.
4. The descriptor for the third cycle in the Framework for Qualifications of the European Higher Education Area agreed by the ministers responsible for higher education at their meeting in Bergen in May 2005 in the framework of the Bologna process corresponds to the learning outcomes for EQF level 8.

Source: http://ec.europa.eu/education/policies/educ/eqf/index_en.html - web page of the European Commission

